

SG-9 — Proton Safety (GUT Gate 10): CLOSURE RESULT — Fable_Version rendered package. Rendered from sg9_PROTON_SAFETY_CLOSURE_RESULT.md ; frozen technical content unchanged by rendering.

SG-9 — Proton Safety (GUT Gate 10): CLOSURE RESULT

Status — binding header. Disposition (current — §4 result + 2026-06-25 second-pass correction):

R1 = **DERIVED-GIVEN-E** (declared-class Wilson zeros) + **AXIOM-CLOSED-d≤7** (dangerous-class completeness, bounded hand-census) + **OPEN / computation-debt** (all-order d>7). The **"-d≤7" tag stands** — an earlier same-day claim that R1 was DERIVED-GIVEN-E *all-order* was a one-rung over-promotion, demoted by independent second-pass audit under the conservative-default rule. New this turn: the **R4' sector projectors are DERIVED-GIVEN-E** (the operator-level orthogonality + completeness the no-mediator identity needs). Full record + the demotion in §0.0. **Gate-level roll-up: OPEN** (unchanged). Open residuals: all-order completeness (computation-debt), R4 full-labeling (AUDIT), STEP4 KK no-go (a strong rep-theory result carried as a named condition), R2 (SHARED-OPEN), R3/M_R (UNKNOWN), lifetime (Diagnostic / observation-pending — permanent endpoint). By *any open piece* ⇒ **OPEN** (rubric), the gate is **OPEN**; the §4 + R4' results are sub-results, not a gate upgrade. **PROMOTIONS: o. All-order completeness (d>7): OPEN / computation-debt** — an unbounded universal negative (A5-precedent logic). The KK-mediator channel is addressed by the STEP4 triality result (carried as a condition); the all-order local-operator completeness is not DERIVED at any order. **Lifetime: Diagnostic only** (no prediction, no frozen bound). **Battle-of-the-Geometries verdict: SHARED-OPEN** (no Fable win; dim-6 BV pressure is shared by every GUT-class geometry). **Label nature:** the genuine new results are the **R4' sector projectors = DERIVED-GIVEN-E** (color-Casimir orthogonality + exhaustive triality completeness) and the **STEP4 KK-mediator triality no-go** (script-checked 0 fails / 2009 checks; DERIVED-GIVEN named inputs; carried as a condition). The dangerous-class completeness remains **AXIOM-CLOSED-d≤7** (bounded hand-census, NOT a machine certificate); all-order (d>7) is **OPEN**. **PROMOTIONS: o.** Frozen branch dcc66f1b2685 / manifest meta a5b1e6f9d951 **READ-ONLY** (untouched throughout). **Honest ceiling:** serious candidate, **NOT validated**. NOT a proof of absolute proton stability; NOT a lifetime prediction; NOT a cross-geometry uniqueness result.

Terminology note. This disposition was historically written *"Closed by Axiom: AXIOM-DANGEROUS-CLASS-COMPLETENESS"*; it is the **same** disposition and the **same** axiom, standardized here to **"Reduced to Axiom: AXIOM-PROTON-DECAY-OPERATOR-COMPLETENESS"** — the verb *"Reduced"* avoids reading as "solved," and the physics name replaces the jargon *"dangerous"* (= the baryon/lepton-number-violating operators that endanger the proton). Quoted text inside the verbatim specialist (§2) and audit (§3) sections retains its original wording.

§0.0 §4 RESULT + SECOND-PASS CORRECTION — 2026-06-25 (current disposition)

This block is the current disposition. It records the §4 KK-overlap analysis and the R4' reconstruction, and it corrects a same-day over-promotion: an earlier version of this block claimed R1-all-order = DERIVED-GIVEN-E (all-order) with the "-d≤7" tag retired. An independent second-pass audit (workflow wwf566161) demoted that by one rung under the conservative-default rule ("when two dispositions are defensible, take the weaker"). The over-promotion is recorded, not hidden. The body (§0–§3, incl. the verbatim §2/§3) is the pre-§4 provenance.

Genuine progress (DERIVED-GIVEN-E — verified, stands):

- R4' sector projectors on the operator-external (zero-mode) bundle: DERIVED-GIVEN-E.** Macro-orthogonality $\Pi_q \Pi_\ell = 0$ is forced by the $SU(3)_c$ Casimir / triality label E carries ($v_3 \otimes v_1 = 0$; eigenprojectors for distinct Casimir eigenvalues $C_2 = 4/3$ vs 0) — independent of the Z_6/Z_2 chamber data, immune to generation mixing. No-orphan completeness $\Pi_q + \Pi_\ell = 1$ is an exhaustive triality partition resting on **three named E-inputs**: the observed color reps $\{3, \bar{3}, 1\}$, the frozen chiral index $\chi = -3$, and the frozen v^c -lepton sector labeling. This upgrades the **macro-orthogonality the no-mediator identity actually needs** from AUDIT → DERIVED-GIVEN-E. (The from-scratch reconstruction of the *full internal-sector* labeling $\Pi_u/\Pi_d/\Pi_e/\Pi_\nu$ stays the **R4 AUDIT** residual.)
- Declared-class dim-6/7 Wilson zeros: DERIVED-GIVEN-E** (zero-mode, declared class) via $\Pi_q \Pi_\ell = 0$.
- STEP4 KK-mediator leptoquark no-go: a strong rep-theory result** — the $K_6 = SU(3)/T^2$ triality selection rule (every KK boson is triality-0 → no leptoquark at any level; charged-bundle Frobenius + triality homomorphism $P/Q \cong \mathbb{Z}_3$; script-checked 0 fails / 2009 checks), **DERIVED-GIVEN** the named structural

inputs (product geometry + descended-content). It closes the **KK-mediator channel** and is **carried as a named condition** — *not* a stand-alone all-order R1 theorem.

4. **Heavy KK color-octet (lepton-harmonic) fermions: confirmed non-orphans** — triality-0 color-adjoint, non-external, loop-only; they don't break completeness or supply a leptoquark.

The corrected ceiling (second-pass demotion under conservative-default):

- **Dangerous-class completeness through $d \leq 7$: AXIOM-CLOSED- $d \leq 7$** (bounded hand-census; NOT a machine certificate; d = operator mass-dimension, never the 13D spacetime dimension). **The " $d \leq 7$ " scope tag stands — its earlier retirement is withdrawn.**
- **All-order ($d > 7$) completeness: OPEN / computation-debt** — an *unbounded* universal negative (same logic as the A5-actor precedent: a bounded census cannot close an unbounded negative). **Not DERIVED at any order.** STEP4 (KK no-go) and the full R4 bundle-labeling travel as named **un-discharged conditions**.
- **R1 overall: DERIVED-GIVEN-E (declared-class zeros) + AXIOM-CLOSED- $d \leq 7$ (completeness) + OPEN/computation-debt (all-order).** NOT "DERIVED-GIVEN-E all-order."

Gate: OPEN (unchanged). Open residuals: all-order completeness (computation-debt), R4 full-labeling (AUDIT), STEP4 KK no-go (carried condition), R2 (SHARED-OPEN), R3/M_R (UNKNOWN), and the **lifetime** (Diagnostic / observation-pending — a *permanent* honest endpoint, no τ_p anchor). **Not a proof of proton stability; no lifetime predicted.**

Provenance + honesty note. Workflows `w12giz239` (§4 KK no-go) + `wwf566i6i` (R4' reconstruction + second-pass demotion). The all-order over-promotion was caught and demoted by the independent second pass under the conservative-default rule — the method working as designed (the campaign's 7th earned demotion).

PROMOTIONS:0; frozen branch `dcc66f1b2685` / `a5b1e6f9d951` **READ-ONLY.**

Reading guide (what this document is, and how to read it)

What this is. This is the **closure-result companion** to the SG-9 attack dossier

(`01_DOSSIER_SG9_PROTON_SAFETY.md`). The attack dossier *names the residuals and the attack plan*; **this document records what happened when the highest-leverage residual (R1) was actually attacked, and whether the result survives an independent adversary.** It is the merged, integrated record of two raw work-products from the `SG9_PROTON_SAFETY_HANDOFF` package:

1. a **specialist closure** that pushed R1 (declared-vs-physical operator class) from "candidate-grade coverage residual" to a bounded **AXIOM-PROTON-DECAY-OPERATOR-COMPLETENESS- $d \leq 7$** by a four-step hand-hardening (axiom statement + $d \leq 7$ operator census + escapee-bin-empty proof + KK-tower no-go); and
2. an **independent refute-before-upgrade audit** (two adversarial verifiers — an SMEFT operator-census re-derivation and a disposition/discipline audit — plus a final verdict) that tried to break the closure, **failed to break it**, found **two non-fatal corpus-editorial defects**, and authorized integration **under five binding conditions**.

What this is NOT. Not a reprint of the attack dossier (that document owns the full residual register R1–R9, the witness ledger, and the per-residual attack plan — referenced here, not duplicated). Not a mutation of the frozen branch. Not a promotion: the gate roll-up stays **OPEN**.

How to read it. - §0 — **Executive summary** (≈ 1 page, **human-readable**). The gate, the verdict, what it means, what stays open, the honest ceiling. Read this if you read nothing else. - §1 — **Final integrated disposition.** The per-half disposition table; the five binding integration conditions (verbatim); the binding caveats; the residuals still open (R1-all-order, R4, STEP4-KK, R2, R3); the two editorial defects with the recommended corpus fix; the integration-ready status text. - §2 — **The specialist closure (full, verbatim-preserving).** The axiom + the four-step hardening, every lemma and table. - §3 — **The independent refute-before-upgrade audit (full, verbatim-preserving).** Census re-check (5 points), disposition + discipline audit (5 kill-tests), final verdict (why it survives, two defects, integration-ready text, five conditions). - §4 — **What hardens it next.** The 13D KK-overlap **all-order** analysis (using the already-computed $K_6/S^2/S^1$ KK+Dirac spectra) as the concrete attack on the all-order OPEN residual; machine-certificate regeneration; the two corpus editorial fixes.

Discipline (carry verbatim). AXIOM-CLOSED $\neq$ proven. given-E $\neq$ derivation of E. selection $\neq$ derivation. dissolved $\neq$ solved. A *selection-rule pass on a declared class* $\neq$ a *proof of absolute proton safety*. The five integration conditions in §1.2 are **binding**; violating any one demotes the disposition back to OPEN.

§0. EXECUTIVE SUMMARY

0.1 The gate

SG-9 (GUT Gate 10) — proton safety, on the **frozen 13D K₆ branch only**. The Standard-Model-routing backbone is a **product** of compact factors,

$$K_{\text{gauge}} = K_6 \times S^2 \times S_Y^1/\mathbb{Z}_2, \quad K_6 = SU(3)/T^2,$$

not an embedding into a single simple GUT group. The zero-mode gauge algebra is the **direct sum** $\mathfrak{su}(3)_c \oplus \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$. Because the geometry is a product, there is **no off-diagonal X/Y heavy gauge boson** connecting quarks to leptons through one multiplet and **no coloured-Higgs triplet** (the Higgs is a Wilson-line $SU(2)_L$ doublet). On top of that absence-of-mediator structure, the tenth load-bearing term $\mathcal{E}_{\text{proton}}$ supplies sector projectors with macro-orthogonality $\Pi_q \Pi_\ell = 0$, which delivers the exact algebraic identity

$$\Pi_q M \Pi_\ell = 0 \quad \text{for every sector-respecting } M,$$

so every Wilson coefficient in the **declared** dangerous class vanishes **identically** — a zero, not a tuned-small number. This is the structural move that dissolves the failure mode that *executed* minimal $SU(5)$ ($\tau_p \sim 10^{30}$ yr vs Super-K's $\gtrsim 10^{34}$ yr).

The dossier's standing status was **PARTIAL** with the central residual **R₁ OPEN**: the projector pass is exact, but only against a **declared** dangerous class $\mathcal{O}_{\text{declared danger}}$. Whether that declared class is **physically complete** — whether every operator the Super-K bound constrains is actually caught — was the gate's own named falsifier, held at **candidate-grade**.

0.2 The verdict

Reduced to Axiom: AXIOM-PROTON-DECAY-OPERATOR-COMPLETENESS-d $\leq$ 7 — UPHOLD on independent refute-before-upgrade audit.

A specialist performed a **bounded operator census through mass dimension 7** on the frozen active-branch field content and proved that, through $d \leq 7$, the **"escapee bin is empty"**: every admissible perturbative, local, SM-zero-mode $\Delta B=1$ nucleon-decay operator through dimension 7 is **either** (a) sector-crossing — carrying one quark-sector leg and one lepton-sector leg — and killed by $\Pi_q M \Pi_\ell = 0$, **or** (b) requires a mediator absent from the product spectrum (X/Y boson, coloured Higgs, scalar leptoquark, or a sector-non-orthogonal KK/chamber mode). This sharpens R₁'s candidate-grade residual into a **named, target-blind, bounded axiom**.

Two independent adversaries then ran the refute-before-upgrade protocol: - **Census re-check (verdict: SOUND)**: an independent SMEFT operator-census re-derivation from first principles (SM Weyl content; exact $SU(3)_c$ / Lorentz/hypercharge invariance), cross-checked against Weinberg 1979, Wilczek–Zee 1979, and Lehman arXiv:1410.4193 (the complete $d=7$ basis). Lemma 1 sound; the $d=6$ set is the complete Weinberg/Wilczek–Zee basis; the $d=7$ set is the complete Lehman basis (all single-field-dressed, all sector-crossing); escapee-bin-empty sound through $d \leq 7$; KK-no-bypass valid as scoped. - **Disposition + discipline audit (verdict: HONEST/UPHELD)**: five kill-tests (overclaim, no-target-loading, reduce-not-relabel, lifetime-kept-Diagnostic / all-order-kept-OPEN, PROMOTIONS: o-compatibility) all pass, with one named caveat (the hand-proof rests on two un-discharged premises — see below). - **Final verdict: UPHELD**. The result survives refutation as an honestly-labeled correct partial result.

The decisive reason it survives where a *prior* attempt failed: the corpus precedent **A₅-actor** was **demoted from AXIOM_CLOSED to OPEN** because its success criterion was a universal negative over an **unbounded** competitor matrix (genuinely unclosable by enumeration). SG-9's universal negative ("the 'other' bin is empty") is **bounded to $d \leq 7$** , and a **finite enumeration genuinely can close a bounded universal negative**. That is the exact discriminator that lets SG-9 reach the ceiling A₅ could not.

0.3 What it means

It means R₁'s **$d \leq 7$ slice is now Reduced to Axiom — the honest ceiling the dossier itself predicted**. The gate's R₁ residual has moved from "a candidate-grade, asserted physical-completeness claim" to **"a named, target-blind, bounded axiom floor on the $d \leq 7$ slice, plus a bounded hand-census that confirms the standard operator types."** The gate itself stays **OPEN** (its other residuals are not yet terminal); what changed is R₁'s $d \leq 7$ slice. This is a **real honesty / reproducibility gain — not a promotion**. The declared-class Wilson zeros remain **DERIVED-GIVEN-E** (an algebraic theorem given the frozen geometry, labeling, and sector-respecting M); the new content is that the *physical-completeness* step is now reduced to one named posit on a bounded slice, rather than left as a bare row-by-row "Covered" assertion.

0.4 What stays open

- **R1-all-order ($d > 7$):** physical completeness for operators of dimension > 7 is **OPEN** / **computation-debt**. The census is bounded; the all-order universal statement is honestly left as the named posit, not proven.
- **Two un-discharged premises inside the hand-proof** (must travel with the label as explicit conditions, not as proven):
- **R4** — bundle-orthogonality $\Pi_\ell \Pi_\ell = 0$ on the active-branch matter bundle is **AUDIT** / **asserted** at A2.6/A2.8, verified only on the frozen labeling, **NOT reconstructed from scratch**. It is the premise of Lemma 3.
- **STEP4 KK-tower no-go** — "no level of the gauge KK tower carries simultaneous colour-**3** + nonzero **T₃** + nonzero **Y_ℓ**" is a **claim**, not a discharged lemma.
- **R2 (SHARED-OPEN):** dim-6 BV suppression is **not a uniqueness** / **Fable win**; the filter-not-determiner Battle verdict is preserved. The bounded-census confidence must **not** inflate this.
- **R3 (M_R / dim-5 Weinberg scale):** the dim-5 Weinberg leg ($\Delta B = 0$, $\Delta L = \pm 2$, $\Lambda = M_R$ UNKNOWN) is correctly kept **SEPARATE** from the $\Delta B=1$ claim. Folding it in would be a smuggle.
- **Lifetime:** stays **Diagnostic only**. No prediction; no frozen bound.

0.5 The honest ceiling

Serious candidate, NOT validated. This is **not** a proof of absolute proton stability — AXIOM-CLOSED means the residual is reduced to exactly one named, unproven, value-free posit; the assumption count does **not** drop. It is **not** a proton-lifetime prediction. It is **not** a regenerated machine certificate — it is a bounded **hand-proof**, and the `G10_proton_safety/` machine cert is a distinct, weaker object that checks only the frozen labeling on 25 randomized members. The gate roll-up stays **OPEN** (does not upgrade). **PROMOTIONS:o; frozen branch READ-ONLY.**

§1. FINAL INTEGRATED DISPOSITION

1.0 How this gate is graded (the rubric)

Per the program's **gate-grading rubric** (`GATE_GRADING_RUBRIC.md`): **a gate's status = its least-closed residual**. A gate is **OPEN** if *any* piece is still open (a computation-debt, an OPEN item, an un-discharged premise, a BLOCKED item, or an unmeasured anchor); it earns a terminal status (**Reduced to Axiom** / **DERIVED**) only when *every* residual is terminal. "PARTIAL" is retired.

This gate is therefore **OPEN**. The operator-safety leg is terminal — **Reduced to Axiom:** `AXIOM-PROTON-DECAY-OPERATOR-COMPLETENESS- $d \leq 7$` — and that is a genuine end-state (AXIOM-anchoring is **terminal and co-equal with DERIVED**: all physics bottoms out at axioms or measured anchors; DERIVED merely anchors to axioms already in the floor). But all-order ($d > 7$) completeness and the lifetime are **computation-debts** — calculations owed, not axioms reached — and two premises (R4, STEP4) are AUDIT/claim. A computation-debt is *not yet* at an axiom, which is the sole honest sense in which this gate is unfinished. When the §4 computations land, those residuals reach a terminal state and the status flips off OPEN.

1.1 Per-half disposition table

This is the honest two-half accounting that both the specialist and the auditors converge on. The gate has always split into a **derivable half** (the declared-class zeros) and an **axiom-closed half** (physical completeness). The $d \leq 7$ hardening sharpens the second half from "candidate-grade asserted" to "AXIOM-CLOSED on a bounded slice, given two AUDIT premises."

Half	Disposition	Reason
Declared-class Wilson zeros	DERIVED-GIVEN-E	Given the frozen geometry, the frozen labeling, and a sector-respecting $\mathbf{M}$, $\Pi_q \mathbf{M} \Pi_q = \mathbf{0}$ is an algebraic theorem (Lemma 2). Not an axiom.
Dangerous-class completeness (through $d \leq 7$) (§4 + 2026-06-25 correction)	AXIOM-CLOSED- $d \leq 7$ (bounded hand-census)	The "- $d \leq 7$ " tag stands — an earlier all-order retirement was withdrawn (demoted by second-pass audit). New: sector projectors $R4' = \text{DERIVED-GIVEN-E}$; declared-class Wilson zeros = DERIVED-GIVEN-E. See §0.0.
All-order completeness ($d > 7$) (§4 + correction)	OPEN / computation-debt	Unbounded universal negative (A5-precedent logic) — not DERIVED at any order. KK-mediator channel addressed by the STEP4 triality result (carried as a condition); local-operator all-order completeness un-discharged.
Proton lifetime	Diagnostic only / measured-but-irreducible boundary + computation-debt	The corpus explicitly claims no hard lifetime; a prediction would need frozen Wilson coefficients, hadronic matrix elements, and RG running, none frozen to certificate standard.
Cross-geometry uniqueness (R2)	SHARED-OPEN (DISCLOSED)	dim-6 BV is a pressure shared by all GUT-class geometries; the pass is a <i>filter</i> , not a <i>determiner</i> . No Fable win.
dim-5 Weinberg / seesaw scale $\mathbf{M}_R$ (R3)	OPEN / UNKNOWN (BG10-FROZEN-MR)	No value, no formula, no hash. Kept separate from the $\Delta B=1$ claim.
Whole SG-9 / Gate 10 roll-up	OPEN	§4 + $R4'$ progress: sector projectors DERIVED-GIVEN-E; declared-class zeros DERIVED-GIVEN-E; completeness AXIOM-CLOSED- $d \leq 7$; STEP4 KK no-go a strong result carried as a condition. Open: all-order (computation-debt), $R4$ full-labeling (AUDIT), $R2$ (SHARED-OPEN), $R3/M_R$ (UNKNOWN), lifetime (Diagnostic). By <i>any open piece</i> $\Rightarrow$ OPEN (rubric), the gate is OPEN — sub-results, not a gate upgrade.

1.2 The FIVE BINDING INTEGRATION CONDITIONS (verbatim — violate any → demote to OPEN)

These are the conditions under which integration was authorized by the final verdict. They are **binding** and must be carried **verbatim** wherever this result is summarized.

1. Carry the "- $d \leq 7$ " scope tag verbatim; never let it read as unqualified AXIOM-PROTON-DECAY-OPERATOR-COMPLETENESS.
2. Carry the two un-discharged premises ($R4$ orthogonality AUDIT; STEP4 KK-tower no-go = claim) as explicit conditions of the hand-proof.
3. Gate roll-up stays PARTIAL; all-order stays OPEN; lifetime stays Diagnostic.
4. Label AXIOM-CLOSED (bounded hand-proof), explicitly "NOT a regenerated machine certificate."
5. Do not let bounded-census confidence inflate the Battle verdict off SHARED-OPEN.

Presentation note on condition 3. We present the gate roll-up as **OPEN**, not the audit's word "PARTIAL." OPEN is *strictly more conservative* (weaker-of-two rule; dossier §A.2 "PARTIAL → conservatively OPEN"; GATE_GRADING_RUBRIC.md), so condition 3's no-promotion intent is honored *a fortiori*. The five conditions above are preserved **verbatim** as the binding audit record.

1.3 Binding caveats (must ship verbatim with the label)

- **NOT a proof of absolute proton stability.** AXIOM-CLOSED = reduced to one named, unproven, value-free posit; the assumption count does not drop (05_METHOD lines 11–14).
- **NOT a proton-lifetime prediction.** Lifetime stays Diagnostic only (L4; ledger line 294).
- **NOT a regenerated machine certificate.** This is a bounded hand-proof; the 610_proton_safety/ machine cert checks only the frozen labeling on 25 randomized members — a distinct, weaker object.
- **PROMOTIONS:o. Frozen branch READ-ONLY.** The gate roll-up stays **OPEN** and does **NOT** upgrade.

1.4 Residuals still open (must travel with the label; demote if any is dropped)

Note: this is a *selected* sub-residual list — the live ones that travel with the $d \leq 7$ label. The **complete residual register R1–R9** (including $R5$ ledger-harness, $R7$ selected-not-forced, $R8 \tau=\omega$) lives in 01_DOSSIER_SG9_PROTON_SAFETY.md ; this table is not the full register.

Residual	Standing	Why it stays open / what it is
R1-all-order	OPEN / computation-debt	Physical completeness for $d > 7$. The census is bounded to $d \leq 7$; the all-order universal statement is the named posit, not a theorem.
R4 (bundle-orthogonality AUDIT)	AUDIT — carried as a condition, not proven	$\Pi_q \Pi_\ell = 0$ on the active-branch matter bundle is asserted at A2.6/A2.8, verified on the frozen labeling only, NOT reconstructed from scratch — a premise of Lemma 3 (02 line 289).
STEP4 KK-tower no-go	CLAIM — carried as a condition, not a discharged lemma	"No level of the gauge KK tower carries simultaneous colour- 3 + nonzero T_3 + nonzero Y_L " (01 line 348). §4 below is the concrete attack on it.
R2 (SHARED-OPEN)	DISCLOSED — preserved	dim-6 BV suppression is not a uniqueness/Fable win; the filter-not-determiner Battle verdict is preserved (the result claims no uniqueness).
R3 (M_R / dim-5 Weinberg)	OPEN / UNKNOWN — kept separate	The dim-5 Weinberg leg ($\Delta B = 0$, $\Delta L = \pm 2$, $\Lambda = M_R$ UNKNOWN) is correctly kept SEPARATE from the $\Delta B = 1$ claim and stays Bounded / M_R -UNKNOWN. Folding it in would be a smuggle.

1.5 The TWO editorial defects found by the audit (corpus-editorial; neither opens an escapee)

The independent census found exactly two presentational defects in the primary corpus. **Both are corpus-editorial; neither opens an escapee or changes any status.** The recommended fix is recorded for the corpus owner.

DEFECT 1 — the "QQQL HH (dim-7)" double-Higgs slip (actually $d=8$). The Appendix L ledger row writes the $d=7$ example as "QQQL HH" — a **double** Higgs insertion. A $d=6$ $\Delta B=1$ core (mass dimension 6) dressed by **two** scalar fields (**HH**, each dimension 1) is **mass dimension 8, not 7**. The genuine $d=7$ dressing is **single-field** (one **H** or one derivative): $d=6$ core $\times$ one scalar = $d=7$. The L.2b coverage table (02 line 318: "QQQL Φ ") and the CR10 companion (03 line 198: "QQQL $\cdot\Phi$ ") **both get this right** with the single-field form. So the manuscript is **internally inconsistent on this one ledger row**. Because **both** the single-H ($d=7$) and double-H ($d=8$) forms retain the quark–lepton crossing, **no escapee is created** and the closure conclusion is unaffected. - **Recommended corpus fix:** relabel the Appendix L ledger row from "QQQL HH (dim-7)" to single-H "QQQL-H (dim-7)" (or, if the double-H form is intended for completeness, move it to a $d=8$ row), so the ledger matches L.2b and CR10. Confirmed at 02 line 224.

DEFECT 2 — the $\bar{d}^c \bar{d}^c \bar{u}^c$ row is not a stand-alone Lorentz scalar. The Appendix L ledger presents the n -side channel $\bar{d}^c \bar{d}^c \bar{u}^c$ (the $\bar{d}^c \bar{d}^c \bar{u}^c$, n - $\bar{n}$ -type, no-lepton, quark-only $\Delta B=1$) as a clean **stand-alone** $\Delta B=1$ operator. But **three Weyl fermions do not form a Lorentz scalar** (three half-integer spins cannot contract to spin 0). It only closes at higher dimension (with an additional fermion or covariant completion) and, even then, it still **requires the absent coloured-triplet mediator** — so it is killed by **Ingredient 1 (coloured-triplet absent)**, **not by the projector identity $\Pi_q M \Pi_\ell = 0$** . This is exactly the quark-only escape route that the census's STEP3 disposes of via Ingredient 1 rather than the projector. The defect is **presentational**: showing it as a clean operator misrepresents *which mechanism* kills it. - **Recommended corpus fix:** footnote the $\bar{d}^c \bar{d}^c \bar{u}^c$ row that three Weyl fermions are not a Lorentz scalar, so the operator closes only at higher dimension, and that it is killed by **Ingredient 1 (absent coloured-triplet mediator)**, not by the projector identity. Confirmed at 02 line 227.

1.6 Integration-ready status text (from the final verdict)

Gate-10 operator-safety status line — change **FROM** "PARTIAL (operator pass; physical completeness CANDIDATE-grade)" **TO**:

Gate 10 — Operator safety: Reduced to Axiom: AXIOM-PROTON-DECAY-OPERATOR-COMPLETENESS ($d \leq 7$ bounded hand-proof). Every admissible perturbative local SM-zero-mode $\Delta B=1$ nucleon-decay operator through mass dimension 7 on the frozen 13D product branch (gauge algebra $\mathfrak{su}(3)_c \oplus \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$, no simple-group embedding) is either (a) sector-crossing — carrying one quark-sector leg and one lepton-sector leg — and killed by the projector identity $\Pi_q M \Pi_\ell = 0$, or (b) requires a mediator absent from the product spectrum (X/Y boson, coloured Higgs, scalar leptoquark, or a sector-non-orthogonal KK/chamber mode). The declared-class Wilson zeros are **DERIVED-GIVEN-E**; the full physical $d \leq 7$ class is **AXIOM-CLOSED** by a bounded hand-census (**NOT** a regenerated machine certificate); all-order ($d > 7$) physical completeness remains **OPEN / computation-debt**. Gate-level roll-up stays **OPEN** (any open piece $\Rightarrow$ OPEN; the operator-safety axiom is a sub-result, not a gate upgrade).

1.7 Convergence note (why both halves of the audit and the final verdict agree)

The specialist's own four-step result and both independent verifiers converge on the **same** disposition, with the same scope and the same two un-discharged premises. The specialist labeled it AXIOM-PROTON-DECAY-OPERATOR-COMPLETENESS- $d \leq 7$, explicitly NOT DERIVED and NOT a machine certificate; the census re-check independently

re-derived the operator content and found the census logic mathematically correct through $d \leq 7$; the disposition audit confirmed it is target-blind, reduce-not-relabel, lifetime-Diagnostic, all-order-OPEN, and PROMOTIONS:o-compatible — flagging only that the two premises (R4, KK-tower) must travel with the label. The final verdict verified every load-bearing anchor against the primary files and returned **UPHELD**. The result is correct, honestly scoped, target-blind, reduce-not-relabel, and does not promote the gate.

§2. THE SPECIALIST CLOSURE (full)

The cold specialist's output from the SG9_PROTON_SAFETY_HANDOFF package: the initial axiom-closure verdict, then the four-step hardening. Preserved in full. The specialist read oo_HANDOFF_README first and treated it as binding; held the discipline throughout (AXIOM-CLOSED ≠ proven; bounded hand-proof, not a regenerated machine certificate; lifetime kept Diagnostic; no target-loading). PROMOTIONS:o; frozen branch read-only.

2.1 PART 1 — Initial verdict

Reduced to Axiom: AXIOM-PROTON-DECAY-OPERATOR-COMPLETENESS.

Axiom statement. On the frozen product-factor 13D branch, with observed SM content E supplied and the C6/B1 admissibility rulebook accepted, every gauge-invariant, admissible perturbative operator that can drive experimentally relevant nucleon decay is either: - **sector-crossing** with one quark-sector leg and one lepton-sector leg, hence lies in $\mathcal{O}^{\text{declared danger}}$ and is killed by $\Pi_q M \Pi_l = 0$; or - **requires** a coloured-triplet / leptoquark / simple-group X/Y-type **mediator that is absent** on the frozen product geometry.

This is **AXIOM-CLOSED**, not **DERIVED**. It names the irreducible debt; it does not pay it. The method file defines AXIOM-CLOSED as one unproven, value-free posit, not a proof.

2.1.1 Coverage verdict on L.2a/L.2b

The declared class covers the standard dangerous perturbative proton-decay classes, but **physical completeness is not proven**. The primary corpus declares the dangerous class before applying the identity (sector-respecting Wilson operators with one quark-sector leg and one lepton-sector leg), lists the standard $d=6$ classes (QQQL, $u^c u^c d^c e$, $QLu^c d^c$, $QQu^c e^c$) and relatives, then states every coefficient in that declared class vanishes by $\Pi_q M \Pi_l = 0$. The coverage table marks the usual $d=6$ channels covered, coloured-Higgs-like channels vacuously covered (coloured Higgs absent), leptoquark-like channels covered by sector orthogonality / absence of a suitable KK mediator, and nonperturbative / Planck-suppressed channels outside scope or diagnostic. **That is enough for an audited assertion, not a theorem of full physical operator completeness.** L.2b is a row-by-row "Covered" assertion, not a coverage theorem.

2.1.2 Honest disposition

Half	Disposition	Reason
Declared-class Wilson zeros	DERIVED-GIVEN-E	Given the frozen geometry, frozen labeling, and sector-respecting M, $\Pi_q M \Pi_l = 0$ is an algebraic theorem.
Full physically dangerous perturbative operator class	AXIOM-CLOSED	Closed only by AXIOM-PROTON-DECAY-OPERATOR-COMPLETENESS; no all-operator theorem supplied.
Physical completeness as theorem	OPEN / computation-debt	Requires a target-blind enumeration of admissible $\Delta B \neq 0$, $\Delta L \neq 0$ operators and no-go lemmas for escape bins.
Proton lifetime	Diagnostic only / measured-but-irreducible boundary + computation-debt	The corpus explicitly claims no hard lifetime; a prediction would need frozen Wilson coefficients, hadronic matrix elements, and RG running.
Whole SG-9 / Gate 10 roll-up	PARTIAL	Strong declared-class operator pass, candidate-grade physical completeness, diagnostic-only lifetime.

2.1.3 No-target-loading self-check

Passes. The axiom can be written **without** knowing Super-K observed no proton decay: it is a structural statement about the admissible operator grammar of a product-factor compactification, not a fitted lifetime statement. But it is close to the residual, so it must stay labeled AXIOM-CLOSED, not "proved" / not DERIVED. It also **barely passes reduce-not-relabel**: "all dangerous operators are killed" would be a relabel-fail; the acceptable version is narrower and structural (it classifies every admissible nucleon-destabilizing perturbative operator into "sector-crossing killed by projector" or "mediator-required but mediator absent").

2.1.4 What hardens it next (the four steps)

1. Enumerate all gauge-invariant admissible $\Delta B \neq 0$ and $\Delta L \neq 0$ operators through at least dimension 7 on the frozen active field content.
2. Classify each as sector-crossing, absent-mediator, outside-scope, or escapee.
3. Prove the "escapee" bin is empty.
4. Separately prove no level of the $\mathcal{E}_{\text{gauge}}$ KK tower carries the leptoquark-like joint quantum numbers needed to bypass $\Pi_q M \Pi_\ell = 0$.

2.2 PART 2 — Four-step hardening

Precision guardrail (specialist's own): this is a **bounded hand proof / operator census, not a regenerated Hilbert-series or CSV certificate**. The dossier says the machine ledger / CSV harness is a referenced artifact and independent regeneration remains an audit item. The result hardens the gate mathematically; it does **not** become a new machine certificate.

2.2.1 Executive verdict

For the perturbative, local, gauge-invariant, SM-zero-mode EFT **through dimension 7**, the escapee bin is empty given the frozen branch, the product gauge geometry, the sector projectors, and the admissibility rule that no exotic non-listed matter/mediator may be added. Upgraded statement:

Reduced to Axiom: AXIOM-PROTON-DECAY-OPERATOR-COMPLETENESS-d $\leq$ 7. On the frozen 13D product branch, every admissible perturbative $\Delta B=1$ proton-decay operator through dimension 7 either contains a quark-lepton sector crossing killed by $\Pi_q M \Pi_\ell = 0$, or requires a mediator absent from the product-geometry spectrum (X/Y, coloured Higgs, scalar leptoquark, or sector-nonorthogonal KK/chamber mode). **AXIOM-CLOSED, not DERIVED** — full physical completeness remains the named residual unless a coverage theorem is supplied.

2.2.2 Step 1 — Bounded operator enumeration through dimension 7

Scope: local perturbative operators from the frozen SM matter content E, the Wilson-line Higgs, gauge/KK actors of $\mathcal{E}_{\text{gauge}}$, the $\mathcal{E}_{\text{proton}}$ projectors. Nonperturbative sphalerons/instantons and Planck-gravity operators are outside the operator-class framework (per Appendix L).

- **Dim ≤ 4 :** no proton-decay operator (renormalizable SM conserves B accidentally; a B-violating renormalizable vertex needs an exotic mediator / field content not admissible on the frozen branch).
- **Dim 5:** Weinberg (LH)(LH)/ Λ — $\Delta B=0$, $\Delta L=2$ — *outside proton safety* (it is a lepton-number / Majorana issue; M_R UNKNOWN/open, cannot close the lifetime). Coloured-Higgs-like $\bar{Q}LH_c/\Lambda$ — **absent mediator** (the active-branch Higgs is a Wilson-line SU(2)_L doublet, not an SU(5)-style coloured-triplet partner).
- **Dim 6 — the complete perturbative $\Delta B=1$ core (the four standard classes):** QQQL, $u^c u^d e^c$, QL $u^d d^c$, QQu e^c — all sector-crossing $\rightarrow$ Wilson coefficients vanish by $\Pi_q M \Pi_\ell = 0$.
- **Dim 7:** the dim-6 $\Delta B=1$ cores dressed by one neutral insertion (Higgs / derivative / admissible neutral dressing) that does not change quark/lepton sector membership: QQQL Φ etc. — still sector-crossing $\rightarrow$ killed. (Lepton-only d=7 LNV: outside proton safety. Quark-only $\Delta B=2$: first appears beyond d=7, d $\geq$ 9.)

So through d $\leq$ 7 every perturbative proton-decay operator is **3 quarks + 1 lepton** ($\pm$ neutral dressing) — a quark-lepton sector crossing.

2.2.3 Step 2 — Classification table

Candidate	Dim	Mechanism required	Classification
QQQL	6	quark–lepton mediator	sector-crossing $\rightarrow$ killed by $\Pi_q M \Pi_\ell = 0$
$u^c u^c d^c e^c$	6	quark–lepton mediator	sector-crossing $\rightarrow$ killed
$QLu^c d^c$	6	quark–lepton mediator	sector-crossing $\rightarrow$ killed
$QQu^c e^c$	6	quark–lepton mediator	sector-crossing $\rightarrow$ killed
QQQL Φ + dressed relatives	7	same core + neutral insertion	sector-crossing $\rightarrow$ killed
X/Y-mediated	6+	simple-group off-diagonal gauge boson	absent mediator
coloured-Higgs-mediated	5/6+	coloured Higgs triplet	absent mediator
scalar-leptoquark-mediated $q M_L Q \ell$	6+	leptoquark mediator	absent / projector-zero
physical KK mediator	6+	nonzero zero-mode–KK–zero-mode cross-sector coupling	KK-number + projector killed
chamber-induced cross-sector mode	any	nonzero $\Pi_q M \Pi_\ell$	projector-zero
EW sphaleron / instanton	nonpert.	gauge-field topology	outside perturbative class
Planck-suppressed QQQL/ M_{Pl}^2	6	quantum gravity	outside scoped-GUT claim
Weinberg LLHH/ Λ	5	Majorana scale M_R	lepton-sector debt, not proton safety
six-quark n – $\bar{n}$ -type	9+	$\Delta B=2$	outside $d \leq 7$

2.2.4 Step 3 — Proof the escapee bin is empty through dimension 7

- **Lemma 1 — a perturbative $\Delta B=1$ proton-decay operator through $d \leq 7$ must contain a quark–lepton sector crossing.** A local Lorentz-scalar fermion operator has an even number of Weyl fermions. $\Delta B=1$ needs a three-quark core; three fermions cannot form a Lorentz scalar, so the minimal completion is $3q+1\ell$ (dimension $4 \times 3/2 = 6$). At $d=7$, a scalar/derivative dressing does not change the fermion-sector fact ($3q+1\ell+\Phi/D$). Hence every perturbative $\Delta B=1$, $d \leq 7$ operator has ≥ 1 quark leg and ≥ 1 lepton leg.
- **Lemma 2 — every quark–lepton crossing is killed on the declared domain.** The frozen E_{proton} macro-projectors $\Pi_q = \Pi_u + \Pi_d + \Pi_{Q_L}$, $\Pi_\ell = \Pi_e + \Pi_\nu + \Pi_{L_L}$ satisfy $\Pi_q \Pi_\ell = 0$; for sector-respecting $M = \sum_i \Pi_i M_i \Pi_i$, $\Pi_q M \Pi_\ell = \sum_i (\Pi_q \Pi_i) M_i (\Pi_i \Pi_\ell) = 0$. (A theorem given the projectors, per the corpus — not an axiom.)
- **Lemma 3 — a non-sector-respecting mediator is absent.** A mediator that avoids Lemma 2 must be a genuine cross-sector carrier: X/Y, coloured Higgs, scalar leptoquark, or sector-mixing KK/chamber mode. Appendix L's mediator inventory: X/Y absent (gauge structure is the product $K_8 \times S^2 \times S_Y^1$, not a simple-group embedding); coloured Higgs absent (Wilson-line doublet); scalar leptoquarks absent; spurious KK / chamber mediators handled by two-channel decoupling + projector-zero.

Therefore $\text{Escapee}_{d \leq 7}^{\text{perturbative, local, SM-zero-mode}} = \emptyset$. This is the hardening result.

2.2.5 Step 4 — No leptoquark KK-tower bypass

Claim: no level of the E_{gauge} KK tower supplies a leptoquark mediator that bypasses $\Pi_q M \Pi_\ell = 0$.

Proof. The frozen gauge actor is the descended gauge bundle over the product gauge geometry; its zero-mode gauge algebra is the direct sum $\mathfrak{su}(3)_c \oplus \mathfrak{su}(2)_L \oplus \mathfrak{u}(1)_Y$, not a simple algebra with off-diagonal quark–lepton generators (the frozen geometry context is explicit: $K_8 = SU(3)/T^2$ routes color, S^2 routes weak, $S_Y^1/\mathbb{Z}_2$ routes hypercharge/chirality — product factors, not a unified simple-group quotient). A KK excitation changes the compact-mode number / mass profile of a gauge field; it does **not** change the gauge representation of the excited generator. So the KK tower of E_{gauge} contains KK copies of adjoint / direct-sum gauge actors, not new bifundamental leptoquark generators. A leptoquark mediator would need to live in $\text{Hom}(V_\ell, V_q)$ with simultaneous color + electroweak/hypercharge leptoquark quantum numbers; Appendix L states the E_{gauge} KK tower produces no coloured-and-leptonic-charged mediator at any level. Gauge-redundant KK components are removed from physical S-matrix elements by BRST/Slavnov–Taylor; physical transverse KK modes are blocked at tree level by KK-number conservation (external zero-modes) and at loop level by projector orthogonality. Hence $\text{KK}(E_{\text{gauge}}) \cap \text{Hom}(V_\ell, V_q) = \emptyset$ on the frozen product branch.

2.2.6 Final status after the four steps

Item	Result
Bounded $d\leq 7$ perturbative proton-decay census	Completed by hand
Escapee bin	Empty within scope
Leptoquark KK-tower bypass	No bypass (product gauge algebra + KK rep inheritance + projector selection)
Declared class vs physical class through $d\leq 7$	Matches for perturbative local proton-decay operators
Operator-level disposition	AXIOM-CLOSED , sharpened as AXIOM-PROTON-DECAY-OPERATOR-COMPLETENESS-$d\leq 7$
Lifetime disposition	Diagnostic only / computation-debt , unchanged
Full all-baryon-physics claim	Not closed
Machine-certificate status	Not regenerated here

Right manuscript-grade statement: SG-9 is hardened from "declared-class pass with coverage residual" to " $d\leq 7$ perturbative proton-decay coverage axiom closed." It is **not** a proof of absolute proton stability, **not** a lifetime prediction, and **not** a cross-geometry uniqueness result.

§3. THE INDEPENDENT REFUTE-BEFORE-UPGRADE AUDIT (full)

Two independent adversarial verifiers (SMEFT operator-census re-derivation + disposition/discipline audit) plus a final verdict. Preserved in full. PROMOTIONS:0; frozen branch read-only.

3.1 A. Independent SMEFT operator-census re-check

Confirmed. The CR10 companion writes the $d=7$ relatives as "QQQL- Φ " (single extra field Φ) — which IS the correct $d=7$ dressing ($d=6$ core $\times$ one scalar). The slip is only in the Appendix L ledger (row "QQQL HH (dim-7)"), where the HH double-insertion would be $d=8$. So the manuscript is internally inconsistent on the $d=7$ example notation, but both forms still carry the quark–lepton crossing, so the conclusion is unaffected.

The independent census is completed across all five points. Assessment below.

3.1.1 Independent SMEFT operator-census assessment of SG-9 / Gate 10

The baryon-violating SMEFT operator content was re-derived from first principles (SM Weyl content; exact $SU(3)_c$ color, Lorentz, and hypercharge invariance) and cross-checked against Weinberg 1979, Wilczek–Zee 1979, and Lehman arXiv:1410.4193 (complete $d=7$ basis). Verdicts point by point:

- (1) Lemma 1 — SOUND.** Every gauge-invariant $\Delta B=1$ operator through $d\leq 7$ is forced to contain a 3-quark color-e core plus ≥ 1 lepton. The chain is exact: $\Delta B=1 \Rightarrow$ net quark number $\pm 3 \Rightarrow$ three (anti)fundamental color indices contracted by $\epsilon_{\{abc\}}$ (the unique minimal nonzero-triality singlet); three Weyl fermions carry half-integer Lorentz spin, so a Lorentz-scalar operator requires an even fermion count, i.e. a 4th fermion; that 4th fermion must be a color singlet to keep the operator colorless, and the only color-singlet fermions are leptons (L, e^c, ν^c). Hence the 4th leg is a lepton — the quark–lepton crossing. The would-be evasion ("a $\Delta B=1$ operator with no lepton") **does not exist at $d\leq 7$** : closing $\epsilon(qqq)$ without a lepton needs another quark (giving non-integer ΔB or pushing to $\Delta B=2, d\geq 9$) — there is no $d\leq 7$ $\Delta B=1$ zero-lepton local scalar. So every $d\leq 7$ $\Delta B=1$ operator genuinely is caught by $\Pi_q M \Pi_\ell = 0$. The dossier itself correctly flags this exact escape route (line 350) and correctly disposes of it via Ingredient 1 (absent colored-triplet), not the projector identity.
- (2) $d=6$ set — COMPLETE.** The four classes $QQQL, u^c u^c d^c e^c, QL u^c d^c, QQ u^c e^c$ are the full $\Delta B\neq 0$ $d=6$ basis (Weinberg/Wilczek-Zee; Warsaw basis). Hypercharge closes on each; all are sector-crossing. Confirmed.
- (3) $d=7$ set — COMPLETE and all sector-crossing, with one notation flaw (non-fatal).** Lehman's complete $d=7$ $\Delta B=1$ operators (LdudH-type, LdddH-type, etc.) all carry exactly one lepton + 3-quark core dressed by a single H/derivative — consistent with Lemma 1, all sector-crossing, all killed by the identity. **FLAW:** the Appendix L ledger writes the $d=7$ example as "QQQL HH" — a **double** Higgs insertion is $d=8$, not $d=7$. The genuine $d=7$ dressing is single-field (the CR10 companion gets this right: "QQQL- Φ "). This is an internal inconsistency / dimension-counting slip in one ledger row, but because both the $d=6$ core and its single- or double-H dressing retain the quark–lepton crossing, **no escapee is created** and the closure conclusion is unaffected.
- (4) Escapee-bin-empty — SOUND through $d\leq 7$.** Follows from (1)+(3): the "other" bin ($\Delta B=1$ operators that are neither sector-crossing nor colored-triplet-mediated) is empty by enumeration through $d=7$. The dossier's labeling is

correct — this is a bounded-dimension result, with the all-order universal statement honestly left as AXIOM-PROTON-DECAY-OPERATOR-COMPLETENESS (a named posit / computation-debt), not as a proven theorem.

(5) KK-no-bypass on the product bundle — VALID as scoped (audited claim, not reconstructed theorem). On the product $K_6 \times S^2 \times S^1 \times Y$ the gauge algebra is $\mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$; KK towers of the gauge sector stay in the adjoint of each factor $[(8,1,0) \oplus (1,3,0) \oplus (1,1,0)]$, so no KK level produces a (3, isospin, Y) leptoquark that could connect qq̄q to a lepton through one propagator. This is genuinely a property a product bundle has and a simple-group embedding lacks, so the structural point is real. The BRST decoupling (Channel A, gauge-redundant = BRST-exact) and the physical-KK selection (Channel B, KK-number at tree + projector orthogonality at loop) are standard and correctly stated — notably the manuscript does NOT overclaim that physical KK modes are BRST-exact. The one residual: the "no joint-quantum-number leptoquark at any KK level" is asserted, not reconstructed from the explicit tower — correctly flagged in the dossier (lines 348, 376) as needing the KK-tower no-go lemma.

Overall. The specialist's verdict — "Reduced to Axiom: AXIOM-PROTON-DECAY-OPERATOR-COMPLETENESS- $d \leq 7$ " — is honest and correctly scoped. The declared-class Wilson zeros are genuinely DERIVED-GIVEN-E; the physical-completeness step is correctly held at AXIOM-CLOSED/CANDIDATE-grade, not promoted to a theorem; the lifetime is correctly Diagnostic-only; the SHARED-OPEN cross-geometry caveat and the M_R -UNKNOWN dim-5 leg are correctly disclosed. The census logic (Lemma 1, escapee-bin-empty, KK-no-bypass) is mathematically correct through $d \leq 7$. The only defect found is a presentational dimension-counting slip ("QQQL HH" labeled $d=7$ in the Appendix L ledger; should be single-H for $d=7$, or relabeled $d=8$), plus the loose presentation of $\bar{d}\bar{d}\bar{u}$ as a clean stand-alone $\Delta B=1$ operator (three Weyl fermions are not a Lorentz scalar; it only closes at higher dimension and still needs the absent colored mediator). Neither defect opens an escapee or changes any status; both are corpus-editorial.

No promotion warranted (none claimed). Frozen branch untouched (read-only throughout). The result survives refutation as an honestly-labeled correct partial result; the two notation defects should be corrected in the ledger but do not undermine the census.

Relevant files: - `.../SG9_PROTON_SAFETY_HANDOFF/01_DOSSIER_SG9_PROTON_SAFETY.md` (escape-hunt table line 344-351; AXIOM-PROTON-DECAY-OPERATOR-COMPLETENESS line 356-365) -

`.../SG9_PROTON_SAFETY_HANDOFF/02_PRIMARY_GUT_proton_safety.md` (Appendix L.2 ledger; the "QQQL HH (dim-7)" slip at the row near line 224; $\bar{d}\bar{d}\bar{u}$ row near line 227) - `.../SG9_PROTON_SAFETY_HANDOFF/03_PRIMARY_CR10_worked_companion.md` (correct single-field $d=7$ dressing "QQQL- Φ " at line 198)

CENSUS VERDICT: SOUND

3.2 B. Disposition + discipline audit

Everything needed is in hand. The closure-campaign result confirms the operative norms precisely: AXIOM_CLOSED $\neq$ proven, "named-axiom escape with a universal-negative obligation = OPEN," and demotion-on-verify is the corpus standard. The A5-actor case is the exact precedent: a named target-blind axiom whose success criterion is an undischarged universal-negative was demoted to OPEN, not accepted as AXIOM-CLOSED. This is directly load-bearing for auditing the SG-9 specialist result.

One final point was verified — that the specialist's $d \leq 7$ bounded census is genuinely consistent with the manuscript's own 11-row ledger and that the dim-5 Weinberg leg is being kept separate (it is — line 218, $\mathbf{A} = \mathbf{M}_R$ UNKNOWN, "Bounded," and the specialist's claim is about $\Delta B=1$, while Weinberg is $\Delta B=0/\Delta L=2$).

DISPOSITION + DISCIPLINE AUDIT — SG-9 specialist result ("Reduced to Axiom: AXIOM-PROTON-DECAY-OPERATOR-COMPLETENESS- $d \leq 7$ ")

SCOPE NOTE. The specialist's returned result is not yet written to disk in the handoff folder (files 00–05 are the inbound package; no specialist-output file exists). The result was audited as stated against (a) the handoff discipline (00_HANDOFF_README, 05_METHOD), (b) the dossier's own residual analysis (01, §3 R1 and §A), (c) the frozen manuscript Appendix L/C10 (02), and (d) the corpus closure norms (CLOSURE_CAMPAIN_RESULT_2026-06-24.md). All paths absolute below.

3.2.1 KILL-TEST 1 — OVERCLAIM (is "AXIOM-CLOSED- $d \leq 7$ " honestly bounded?)

PASS, conditionally. The verdict is explicitly bounded three ways that matter: (i) it is "Reduced to Axiom," which per 05_METHOD line 11-14 and the README BINDING RULES means "reduced to one named, unproven, value-free posit; assumption count does NOT drop; the gap stays open" — i.e. AXIOM-CLOSED is definitionally NOT a proof, NOT DERIVED, and NOT absolute proton stability. (ii) The dimension bound " $d \leq 7$ " is honest about being a *bounded* census, not all-order — the prompt explicitly keeps all-order completeness OPEN/computation-debt. (iii) The "bounded hand proof, explicitly NOT a regenerated machine certificate" caveat is present and is the correct caveat — it matches the dossier's W3/W4 distinction (the `G10_proton_safety/` machine cert checks only the frozen labeling on 25 randomized members; a from-scratch census is a different, hand-level object). No lifetime number is asserted; this is consistent with manuscript L.4 ("Diagnostic only") and ledger line 294. No drift toward "proves the proton is stable" is present in the stated verdict. The one residual overclaim *risk* (not a violation as stated): the label

"AXIOM-CLOSED" for the *full physical $d \leq 7$ class* is stronger than the dossier's own standing, which holds R1 at OPEN with AXIOM-CLOSED only as the "realistic ceiling" (01 lines 290, 384, 697, 714). The specialist reaching the ceiling is permissible IF the universal-negative is genuinely discharged by the bounded census — see Kill-test 3, which is where the real tension sits.

3.2.2 KILL-TEST 2 — NO-TARGET-LOADING (writable without knowing Super-K saw no decay?)

PASS. AXIOM-PROTON-DECAY-OPERATOR-COMPLETENESS as stated is a statement about *operator structure under admissibility* ("every admissible perturbative local SM-zero-mode $\Delta B=1$ operator through dim 7 is sector-crossing or needs an absent mediator") — it contains no proton-lifetime number, no rate, no Super-K bound. It is the same target-blind form the dossier already vetted as kill-test-clean (01 lines 356-365: "writable with no proton-lifetime number in sight ... PASSES the κ^3/π kill-test"). Critically, the four-step hand-proof's logic (Lemma 1: a Lorentz-scalar $\Delta B=1$ operator needs a 3-quark core + a lepton $\Rightarrow$ quark-lepton crossing; Lemma 2: projector orthogonality $\Rightarrow \Pi_q M \Pi_l = 0$; Lemma 3: cross-sector mediator absent on the product spectrum) is derived from the gauge structure and Lorentz invariance, not from the observation that the proton lives. The census would be written identically in a world where Super-K had seen a decay (it would then yield a *falsifier* — an escapee in the "other" bin — which is exactly the good-negative outcome the dossier names at 01 line 387). VERDICT: no target-loading.

3.2.3 KILL-TEST 3 — REDUCE-NOT-RELABEL (genuinely deeper axiom, vs the "all dangerous operators killed" relabel the specialist correctly rejected?)

PASS WITH A NAMED CAVEAT — and this is the load-bearing finding. The specialist did the right thing by NOT accepting the relabel "all dangerous operators are killed" (which would be RELABEL_FAIL per 05 line 61 — the unknown wearing a new name). Instead the accepted axiom is narrower and structural: dangerous $\Delta B=1$ operators are *sector-crossing* (a Lorentz + gauge-rep claim), which is genuinely deeper than "they vanish" because it explains *why* via $\Pi_q \Pi_l = 0$ + mediator absence. That is a real reduction.

HOWEVER — the corpus's own precedent (A5-actor, CLOSURE_CAMPAIGN_RESULT_2026-06-24.md line 36) demoted an exactly-analogous result from AXIOM_CLOSED to OPEN because "the success criterion is a UNIVERSAL NEGATIVE — 'no admissible competitor exists' — which is UNMET." The SG-9 STEP3 "escapee-bin-empty" claim is structurally the same universal negative ("the 'other' bin is empty"). The specialist discharges it *within $d \leq 7$* by a bounded finite census (legitimate — a finite enumeration genuinely can close a bounded universal negative, unlike the unbounded A5 competitor matrix), but two sub-claims inside that census remain non-theorems by the dossier's own accounting and the manuscript's own falsifier: - **Lemma 3** (cross-sector mediator absent on product geometry) depends on projector orthogonality on the *active-branch matter bundle*, which is AUDIT/asserted at A2.6/A2.8, verified only on the frozen labeling, NOT reconstructed from scratch (01 R4, lines 246-248, 472-477). The hand-proof's premise is therefore itself axiom-grade, not machine-real. - **STEP4** (no leptoquark-KK-tower bypass) is flagged in the dossier as "covered (claim; needs KK-tower theorem)" (01 line 348) — a claim, not a discharged lemma.

This means the honest disposition of the *physical-completeness* sub-claim is AXIOM-CLOSED resting on TWO further un-discharged premises (R4 orthogonality + KK-tower no-go), i.e. it is AXIOM-CLOSED-on- $d \leq 7$ given those two AUDIT premises — defensible as a bounded hand-proof, but it must not be read as having *eliminated* the universal negative the way a machine-checked census would. Per 05 line 44 ("when two dispositions are defensible, take the weaker one"), the safe label for the *all-dimension* gate stays OPEN, and AXIOM-PROTON-DECAY-OPERATOR-COMPLETENESS- $d \leq 7$ is the named ceiling for the bounded slice only. The specialist result, as stated, respects this (it labels full physical $d \leq 7$ as AXIOM-CLOSED and all-order as OPEN), so it is REDUCE not RELABEL — provided the $d \leq 7$ scope tag and the two AUDIT premises (R4, KK-tower) travel with the label and are not silently dropped on integration.

3.2.4 KILL-TEST 4 — lifetime kept Diagnostic, all-order completeness kept OPEN?

PASS. The result keeps lifetime as Diagnostic (matches L.4, ledger line 294, dossier R6) and explicitly keeps all-order completeness OPEN/computation-debt. The dim-5 Weinberg leg is correctly NOT swept into the $\Delta B=1$ claim — the specialist's class is $\Delta B=1$ proton decay; Weinberg is $\Delta B=0$, $\Delta L=\pm 2$ with $\mathbf{A} = \mathbf{M}_R$ UNKNOWN (02 line 218; 01 R3). Keeping these separate is correct discipline; folding Weinberg/ $\mathbf{M}_R$ in would have been a smuggle. No layer-smuggling detected: the $\times$ -layer product-geometry fact (no X/Y, no coloured Higgs), the $\oplus$ -layer admissibility rulebook (C6/B1), and the $\otimes$ -layer projectors/Hilbert objects are kept in their correct layers.

3.2.5 KILL-TEST 5 — consistent with PROMOTIONS:0 + the dossier PARTIAL roll-up?

PASS, IF integrated as a sub-result. The gate-level roll-up MUST remain PARTIAL (01 line 30; §A.2 "PARTIAL $\rightarrow$ conservatively OPEN"). AXIOM-CLOSED on R1's $d \leq 7$ slice does NOT promote the gate: the dossier already names AXIOM-CLOSED as the *ceiling per residual*, not a gate upgrade (01 lines 668-670: "would move from PARTIAL-with-CANDIDATE-grade-asserted-status to PARTIAL-with-... a named axiom floor ... a real honesty/reproducibility gain, NOT a promotion"). The specialist result is a *naming* of R1's irreducible posit plus a bounded hand-census that

confirms the standard types — exactly the "AXIOM-CLOSED likely" outcome predicted at 01 line 383. It is PROMOTIONS:o-compatible so long as: (a) the gate stays PARTIAL; (b) R3 ($\mathcal{M}_R$), R4 (orthogonality AUDIT), R5 (ledger harness), R7 (selected-not-forced), R8 ($\tau = \omega$) all stay at their current standing; (c) the SHARED-OPEN / filter-not-determiner Battle verdict (R2) is preserved — the result does NOT claim uniqueness, so this holds.

3.2.6 INTEGRATION CONDITIONS (must all hold, else demote)

1. Carry the " $d \leq 7$ " scope tag verbatim; never drop it to read "AXIOM-PROTON-DECAY-OPERATOR-COMPLETENESS" unqualified.
2. Carry the two un-discharged premises (R4 bundle-orthogonality AUDIT; STEP4 KK-tower no-go = claim) as explicit conditions of the hand-proof, not as proven.
3. Gate roll-up stays PARTIAL; all-order completeness stays OPEN; lifetime stays Diagnostic.
4. Label it AXIOM-CLOSED (bounded hand-proof), explicitly "NOT a regenerated machine certificate" — the caveat the specialist already states.
5. Do not let the bounded-census confidence inflate the Battle verdict off SHARED-OPEN.

Relevant files: .../01_DOSSIER_SG9_PROTON_SAFETY.md (R1 §3.1/§4.1, §A); .../02_PRIMARY_GUT_proton_safety.md (L.2a/L.2b lines 239-329, dim-7 ledger 213-225, falsifier 280/288); .../05_METHOD_AXIOM_CLOSURE_AND_DISCIPLINE.md (AXIOM-CLOSED $\neq$ proven; weaker-of-two rule); .../CLOSURE_CAMPAGN_RESULT_2026-06-24.md (A5-actor universal-negative demotion precedent, line 36; AXIOM_CLOSED semantics).

The specialist result is an honest, target-blind, correctly-scoped AXIOM-CLOSED reduction of R1's $d \leq 7$ slice that does not promote the gate, keeps lifetime Diagnostic, keeps all-order OPEN, and is reduce-not-relabel — its only exposure is that two premises inside the hand-proof (R4 orthogonality, KK-tower no-go) remain AUDIT/claim and must travel with the label.

DISPOSITION VERDICT: HONEST/UPHELD

3.3 C. Final verdict (refute-before-upgrade)

All anchors are verified independently. The critical findings:

1. **Census soundness** — The dossier's own escape-hunt table (01 lines 344-351) is exactly as both assessments describe: the dim-6 four classes are caught, the colored-Higgs and leptquark rows are caught by mediator-absence/KK-claim, and the genuine open escape route is the quark-only/lepton-only "other" bin (line 350), which the specialist's STEP3 empties by finite census.
2. **The notation slip is real and non-fatal** — Appendix L ledger (02 line 224) does write "QQQL HH (dim-7)" with a double-H, which is genuinely $d=8$. The L.2b table (02 line 318) and the CR10 companion (03 line 198) both correctly use single-field "QQQL- Φ " / "QQQL Φ ". So the manuscript is internally inconsistent on this one row, but both forms carry the quark-lepton crossing, so no escapee is created. Confirmed corpus-editorial.
3. **The A5-actor precedent is exactly load-bearing** — CLOSURE_CAMPAGN line 36 confirms: a named target-blind axiom whose success criterion is an *undischarged universal negative* was DEMOTED from AXIOM_CLOSED to OPEN. The decisive distinction the disposition audit drew is correct: A5's universal negative was over an *unbounded* competitor matrix (genuinely unclosable by enumeration), whereas SG-9's universal negative is bounded to $d \leq 7$ (a finite enumeration genuinely can close a bounded universal negative). This is the precise reason SG-9 survives where A5 fell.
4. **The two un-discharged premises are real** — Lemma 3 (R4 bundle-orthogonality) is AUDIT/asserted (02 line 289: "asserted at A2.6/A2.8"), and STEP4 KK-tower no-go is a "claim" (01 line 348). Both must travel with the label.
5. **METHOD discipline confirms** — AXIOM-CLOSED $\neq$ proof (05 line 11-14); weaker-of-two rule (05 line 44); the gate's own honest ceiling is "AXIOM-CLOSED (likely)" (01 line 382-384), which is exactly what the specialist reached.

Both assessments converge, and the independent read of the primary files confirms them. The result is correct, honestly scoped, target-blind, reduce-not-relabel, and does not promote the gate.

VERDICT: UPHELD

The SG-9 specialist result survives refutation. It is a correct, target-blind, honestly-bounded AXIOM-CLOSED reduction of residual R1's $d \leq 7$ slice. Every load-bearing anchor was independently verified against the primary files; both the census re-check and the disposition audit hold. Integration is authorized under the conditions below. PROMOTIONS:o. Frozen branch untouched (read-only throughout).

WHY IT SURVIVES (the three kill-tests that mattered): - **No-target-loading:** AXIOM-PROTON-DECAY-OPERATOR-COMPLETENESS- $d \leq 7$ is a statement about operator structure under admissibility — no proton-lifetime number, no rate, no Super-K bound. Writable identically in a world where Super-K had seen a decay (it would then yield a falsifier in the "other" bin). The dossier already vetted this exact form as κ^3/π -clean (01 lines 356-365). - **Reduce-not-relabel:** the specialist correctly rejected the equal-strength relabel "all dangerous operators are killed" (which would be RELABEL_FAIL per 05 line 41) and instead reduced to the deeper structural claim that dangerous $\Delta B=1$ operators are sector-crossing (a Lorentz + gauge-rep fact), explaining WHY via $\Pi_q M \Pi_l = 0$ + mediator absence. - **The A5-actor demotion does NOT apply.** A5 was demoted (CLOSURE_CAMPAIGN line 36) because its universal negative ranged over an UNBOUNDED competitor matrix. SG-9's universal negative ("the 'other' bin is empty") is BOUNDED to $d \leq 7$, and a finite enumeration genuinely closes a bounded universal negative. This is the exact discriminator that lets SG-9 reach the ceiling A5 could not.

Census logic is mathematically correct through $d \leq 7$: Lemma 1 ($\Delta B=1$ Lorentz-scalar needs a 3-quark color- ϵ core + ≥ 1 color-singlet fermion = a lepton $\Rightarrow$ quark-lepton crossing), the dim-6 set is the complete Weinberg/Wilczek-Zee basis, the dim-7 set is the complete Lehman basis (all single-field-dressed, all sector-crossing), and the escapee bin is empty by enumeration. The dossier's own escape-hunt table (01 lines 344-351) names exactly this open route (line 350) and the census empties it.

TWO DEFECTS FOUND — both corpus-editorial, neither opens an escapee: 1. Appendix L ledger row "QQQL HH (dim-7)" (02 line 224) is a dimension-counting slip — a double-Higgs insertion is $d=8$, not $d=7$. The genuine $d=7$ dressing is single-field, and both L.2b (02 line 318: "QQQL Φ ") and the CR10 companion (03 line 198: "QQQL Φ ") get it right. Internal inconsistency in one row; both forms retain the quark-lepton crossing, so no escapee created. 2. The $\bar{d}\bar{d}\bar{u}$ row (02 line 227) is presented as a clean stand-alone $\Delta B=1$ operator, but three Weyl fermions are not a Lorentz scalar — it only closes at higher dimension and still requires the absent colored-triplet mediator (killed by Ingredient 1, not the projector identity).

Recommended fix: relabel the ledger row to single-H $d=7$ (or move to $d=8$), and footnote the $\bar{d}\bar{d}\bar{u}$ row that it closes only at higher dimension. Conclusion unaffected.

INTEGRATION-READY TEXT (SG-9 dossier R1 + gate status):

Gate-10 operator-safety status line — change FROM "PARTIAL (operator pass; physical completeness CANDIDATE-grade)" TO:

Gate 10 — Operator safety: Reduced to Axiom: AXIOM-PROTON-DECAY-OPERATOR-COMPLETENESS ($d \leq 7$ bounded hand-proof). Every admissible perturbative local SM-zero-mode $\Delta B=1$ nucleon-decay operator through mass dimension 7 on the frozen 13D product branch (gauge algebra $\text{su}(3)_c \oplus \text{su}(2)_L \oplus \text{u}(1)_Y$, no simple-group embedding) is either (a) sector-crossing — carrying one quark-sector leg and one lepton-sector leg — and killed by the projector identity $\Pi_q M \Pi_l = 0$, or (b) requires a mediator absent from the product spectrum (X/Y boson, colored Higgs, scalar leptoquark, or a sector-non-orthogonal KK/chamber mode). The declared-class Wilson zeros are DERIVED-GIVEN-E; the full physical $d \leq 7$ class is AXIOM-CLOSED by a bounded hand-census (NOT a regenerated machine certificate); all-order ($d > 7$) physical completeness remains OPEN/computation-debt. Gate-level roll-up stays PARTIAL.

Binding caveats (must ship verbatim with the label): - NOT a proof of absolute proton stability. AXIOM-CLOSED = reduced to one named, unproven, value-free posit; the assumption count does not drop (05 lines 11-14). - NOT a proton-lifetime prediction. Lifetime stays Diagnostic only (L.4; ledger line 294). - NOT a regenerated machine certificate. This is a bounded hand-proof; the `G10_proton_safety/` machine cert checks only the frozen labeling on 25 randomized members — a distinct, weaker object. - PROMOTIONS:0. Frozen branch READ-ONLY. The gate roll-up does NOT upgrade off PARTIAL.

RESIDUALS STILL OPEN (must travel with the label; demote if any is dropped): - **R1-all-order:** physical completeness for $d > 7$ — OPEN/computation-debt. - **R4:** bundle-orthogonality $\Pi_q \Pi_l = 0$ on the active-branch matter bundle is AUDIT/asserted at A2.6/A2.8, verified on the frozen labeling only, NOT reconstructed from scratch — a premise of Lemma 3, carried as a condition, not proven (02 line 289). - **STEP4 KK-tower no-go:** "no level of the gauge KK tower carries simultaneous color-3 + nonzero T_3 + nonzero Y_l " is a claim, not a discharged lemma (01 line 348) — carried as a condition. - **R2 (SHARED-OPEN):** dim-6 BV suppression is not a uniqueness/Fable win; the filter-not-determiner Battle verdict is preserved (result claims no uniqueness). - **R3:** the dim-5 Weinberg leg ($\Delta B=0$, $\Delta L=\pm 2$, $\Lambda=M_R$ UNKNOWN) is correctly kept SEPARATE from the $\Delta B=1$ claim and stays Bounded/ M_R -UNKNOWN. Folding it in would be a smuggle.

FIVE INTEGRATION CONDITIONS (violate any $\rightarrow$ demote to OPEN): 1. Carry the " $d \leq 7$ " scope tag verbatim; never let it read as unqualified AXIOM-PROTON-DECAY-OPERATOR-COMPLETENESS. 2. Carry the two un-discharged premises (R4 orthogonality AUDIT; STEP4 KK-tower no-go = claim) as explicit conditions of the hand-proof. 3. Gate roll-up stays PARTIAL; all-order stays OPEN; lifetime stays Diagnostic. 4. Label AXIOM-

CLOSED (bounded hand-proof), explicitly "NOT a regenerated machine certificate." 5. Do not let bounded-census confidence inflate the Battle verdict off SHARED-OPEN.

Relevant files (all absolute, verified this session): - C:/Users/cberg/Desktop/VS Code Projects

Random/physics_Journal_and_patents/Fable_Version/rendered/TOE/PER_GATE_DOSSIERS/SG9_PROTON_SAFETY_HANDOFF/01_DOSSIER_SG9_PROTON_SAF (escape-hunt table lines 344-351; AXIOM-CLOSED ceiling lines 382-384; KK-tower claim line 348) -

C:/Users/cberg/Desktop/VS Code Projects

Random/physics_Journal_and_patents/Fable_Version/rendered/TOE/PER_GATE_DOSSIERS/SG9_PROTON_SAFETY_HANDOFF/02_PRIMARY_GUT_proton_saf (DEFECT 1: ledger line 224 "QQQL HH (dim-7)" → d=8; DEFECT 2: $\vec{d}\vec{d}\vec{u}$ line 227; R4 orthogonality asserted line

289; L.2b correct "QQQLΦ" line 318) - C:/Users/cberg/Desktop/VS Code Projects

Random/physics_Journal_and_patents/Fable_Version/rendered/TOE/PER_GATE_DOSSIERS/SG9_PROTON_SAFETY_HANDOFF/03_PRIMARY_CR10_worked_co (correct single-field d=7 "QQQLΦ" line 198) - C:/Users/cberg/Desktop/VS Code Projects

Random/physics_Journal_and_patents/Fable_Version/rendered/TOE/PER_GATE_DOSSIERS/SG9_PROTON_SAFETY_HANDOFF/05_METHOD_AXIOM_CLOSURE_A (AXIOM-CLOSED ≠ proof lines 11-14; weaker-of-two rule line 44) - C:/Users/cberg/Desktop/VS Code Projects

Random/physics_Journal_and_patents/Fable_Version/rendered/TOE/CLOSURE_CAMPAIN_RESULT_2026-06-24.md (A5-actor universal-negative demotion precedent, line 36)

DISPOSITION: Reduced to Axiom: AXIOM-PROTON-DECAY-OPERATOR-COMPLETENESS ($d \leq 7$ bounded hand-proof). Gate roll-up PARTIAL. PROMOTIONS:0. Correct partial result, honestly labeled — integrate with the five conditions and fix the two editorial defects.

§4. WHAT HARDENS IT NEXT

This section is the concrete work program that would push the two **un-discharged premises** (STEP4 KK-tower no-go; R4 bundle-orthogonality) and the **all-order OPEN residual** (R1-all-order, $d > 7$) closer to closure, plus the two corpus editorial fixes. **Nothing here is applied; PROMOTIONS:0; the frozen branch is READ-ONLY.** Each item is a SANDBOX proposal, adversarially tested, not an edit to the frozen branch.

4.1 The 13D KK-overlap all-order analysis — the concrete attack on STEP4 + R1-all-order

This is the single highest-value hardening move. The STEP4 KK-tower no-go is currently a **claim** ("no level of the gauge KK tower carries simultaneous colour-**3** + nonzero T_3 + nonzero Y_L "), and R1-all-order ($d > 7$) is OPEN. **Both can be attacked, all-order in the KK number, using the already-computed $K_6/S^2/S^1$ KK + Dirac spectra** — these spectra were computed pure-geometry, target-blind (the threshold packet δ does not appear in them), and are on disk at .../rendered/TOE/SPECTRA_COMPUTATION_2026-06-24/ (spectrum_K6.md/.csv , spectrum_S2.md/.csv , spectrum_S1.md/.csv). The structural point STEP4 needs is **already visible in those spectra**; the work is to turn the visible structure into a discharged lemma.

Status of STEP4: a strong rep-theory result, carried as a named condition (see §0.0; 2026-06-25 second-pass correction). The route sketched in §4.1 below was executed and adversarially verified (workflow w12giz239 ; rep-theory script-checked 0 fails / 2009 checks): the triality selection rule on $K_6 = SU(3)/T^2$ makes the **KK-mediator no-go a theorem, all-order in the KK number, DERIVED-GIVEN** (i) product / non-color-mixing geometry and (ii) descended-content-only. **But this closes only the KK-mediator channel** — an earlier same-day claim that it (with a dimension-independent Lemma 1) discharged the *whole* all-order R1 was a one-rung over-promotion, demoted by the second-pass audit (wwf566161): all-order ($d > 7$) **local-operator completeness** is an unbounded universal negative, **OPEN / computation-debt**. STEP4 is therefore **carried as a named condition**, not a stand-alone all-order R1 theorem. It does **not** discharge R4 and does **not** upgrade the gate (OPEN). PROMOTIONS:0.

4.1.1 The key structural fact (why a leptoquark KK mediator cannot exist on this geometry)

A leptoquark mediator that could bypass $\Pi_q M \Pi_L = 0$ would have to be a single field carrying **simultaneously** (i) a colour index that connects to the quark ϵ_{abc} core — i.e. it must transform in a **non-trivial-triality $SU(3)_c$** representation (the fundamental **3** / $\bar{3}$, or any triality- ± 1 irrep), so that one propagator can carry colour off the three-quark core onto a colour-singlet lepton — **and** (ii) nonzero weak isospin T_3 and lepton hypercharge Y_L . The decisive observation: **on the product geometry, the KK tower of each factor inherits the representation content of that factor's L^2 decomposition, and the color factor $K_6 = SU(3)/T^2$ admits no triality- ± 1 KK level at all.**

This is not an assertion — it is the **Frobenius selection rule on the flag manifold**, and it is already computed and self-checked in spectrum_K6.md :

"Triality (the $SU(3)$ center $\mathbb{Z}_3$ charge $(p - q) \bmod 3$) is the selection rule: **only triality-0 irreps have a T^2 -fixed vector**, so only they appear in $L^2(SU(3)/T)$. The fundamentals $\mathbf{3}(1, 0)$ and $\bar{\mathbf{3}}(0, 1)$ correctly have $m_0 = 0$ and are **absent** from the scalar spectrum — the hallmark of the flag coset."

The K_6 scalar Laplacian spectrum (eigenvalue = $C_2(\lambda)$, multiplicity = zero-weight multiplicity $m_0(p, q) = \min(p, q) + 1$ if $3 \mid (p - q)$, else 0) runs over **triality-0 irreps only**: $(0, 0)$ the singlet, $(1, 1)$ the adjoint $\mathbf{8}$, $(0, 3)/(3, 0)$ the $\mathbf{10}/\bar{\mathbf{10}}$, $(2, 2)$ the $\mathbf{27}$, and so on. **Every KK level of a field propagating on K_6 sits in a triality-0 $SU(3)_c$ representation** — because a T^2 -fixed vector (the condition for the mode to survive on the coset) exists only for triality-0 irreps. A colour- $\mathbf{3}$ (triality +1) leptoquark KK mode therefore **cannot exist at any KK level** on K_6 : it has $m_0 = 0$, i.e. it does not appear in $L^2(K_6)$ at all. This is the all-order statement STEP4 needs, and it **follows from** the flag-coset Peter–Weyl/Frobenius selection rule at every KK level, not just the lowest — but as a **proposed** discharge (see the status stamp above) it must still be written up and machine-checked (§4.1.2–4.1.3) before STEP4 is reclassified from *claim* to *theorem*.

The same inheritance holds for the other two factors, and confirms there is no escape through the weak or hypercharge towers: - $S^2 = SU(2)/U(1)$ (spectrum_52.md): the scalar tower is ordinary spherical harmonics $l(l+1)$, multiplicity $2l+1$; every KK level sits in an integer-spin $SU(2)_L$ irrep (the coset Frobenius rule keeps only integer $j = l$). The monopole/Dirac ladders for charge $q \in \{0, 1, 2\}$ give $|q|$ zero modes and excited levels $|D| = \sqrt{n(n+|q|)}/R$ with multiplicity $2n+|q|$ per sign. **No KK level changes the $SU(3)_c$ colour of the mode** — S^2 carries weak isospin, not colour. - S^1_Y/Z_2 (spectrum_51.md): the surviving KK tower is $\lambda_n = (n/R)^2$, $m_n = |n|/R$ with $1/R_0 = 6.2814 \times 10^{16}$ GeV; the $\mathbb{Z}_2$ orbifold keeps one chiral Weyl zero mode and makes every $n \geq 1$ level a **vector-like Dirac** (L from the cos/Neumann tower + R from the sin/Dirichlet tower at the same mass). **No KK level changes colour or generates a chiral mirror**. The hypercharge tower carries $U(1)_Y$ charge, not colour.

Putting the three factors together: a KK excitation on the product $K_6 \times S^2 \times S^1_Y/Z_2$ is labeled by $(K_6 \text{ level}; S^2 \text{ level}; S^1 \text{ level})$, and its $SU(3)_c \times SU(2)_L \times U(1)_Y$ representation is the **tensor of the three factors' inherited representations** — which is **(triality-0 colour) $\otimes$ ($SU(2)_L$ irrep) $\otimes$ ($U(1)_Y$ charge)**. The colour factor is **triality-0 at every level**. A leptoquark needs triality ± 1 colour. Therefore:

$$KK(E_{\text{gauge}}) \cap \text{Hom}(V_l, V_q) = \emptyset \quad \text{at every KK level, all-order in the KK number.}$$

This is exactly the "no joint-quantum-number leptoquark at any KK level" statement that the census re-check flagged as **"asserted, not reconstructed from the explicit tower"** (audit point (5)). The $K_6/S^2/S^1$ spectra **are** the explicit tower, and the Frobenius triality selection rule **is** the reconstruction. Writing this up as a formal lemma — "the KK-number-graded $SU(3)_c$ content of any field on $K_6 = SU(3)/T^2$ is triality-0 at every level, hence no leptoquark generator appears at any KK level" — would **discharge STEP4 from a claim into a theorem**, and it would do so **all-order in the KK number** (because the Frobenius rule is a statement about every irrep in L^2 , not just the lowest), directly attacking the all-order residual on the KK channel.

4.1.2 What this does and does not close

- **Does (if written up + machine-checked):** discharge the STEP4 KK-tower no-go (integration condition 2's second premise) into a reconstructed theorem; attack R1-all-order on the **KK-mediator channel** all-order in the KK number (the colour-triality argument is level-independent). This converts one of the two "carried-as-condition" premises into a discharged lemma.
- **Does NOT:** close R1-all-order **as a whole**. The all-order residual also includes the **local operator dimension** axis (operators of mass dimension $d > 7$ built from zero modes), which is a *separate* universal negative not addressed by the KK-tower argument. The KK-overlap analysis closes the **KK-mediator** sub-question all-order; the **higher-dimension local operator** sub-question stays OPEN/computation-debt. Conservative disposition (weaker-of-two rule): even with the KK lemma discharged, R1-all-order stays **OPEN**, and so the gate stays **OPEN**. The win is precision (one of two premises discharged), not promotion.
- **Caveat (carry it):** the $K_6/S^2/S^1$ spectra are **pure-geometry scalar + Dirac spectral building blocks**. The Kostant curvature-endomorphism refinement to the fermion rows $(-R/4 + \dots)$ is noted as the one genuine open refinement in spectrum_K6.md — it shifts fermion rows at $O(1)$ but **does not change the triality content** (it shifts eigenvalues, not representations), so it does not affect the colour-triality no-go. The no-go rests on the representation labels, which are refinement-independent.

4.1.3 The bounded computational task (specialist hand-off)

Hand to the geometry/spectral specialist: **"Using the frozen $K_6/S^2/S^1$ KK + Dirac spectra (already computed, target-blind, on disk), prove the KK-number-graded $SU(3)_c$ representation content of E_{gauge} on $K_6 = SU(3)/T^2$ is triality-0 at every KK level (Frobenius/Peter–Weyl selection rule), hence $KK(E_{\text{gauge}}) \cap \text{Hom}(V_l, V_q) = \emptyset$ all-order in the KK number; report whether this discharges STEP4 into**

a theorem or leaves a residual." This is a **bounded, finite-per-level, all-order-by-selection-rule** representation-theory task — not an open-ended search. The κ^3/π kill-test binds: the lemma must be writable with no proton-lifetime number in sight (it is — it is a statement about coset representation theory), and it must not enlarge the dangerous class to fit whatever vanishes.

4.2 Machine-certificate regeneration (R4 + R5)

The result is explicitly a **bounded hand-proof, NOT a regenerated machine certificate** (integration condition 4). Two reproducibility tasks would convert the hand-proof's *premises* into machine-real objects:

- **R4 — bundle-orthogonality reconstruction (discharges Lemma 3's premise).** Reconstruct the active-branch matter bundle $\mathcal{E}_{\text{matter}}$ **target-blind** from the A2.3 matter-bundle factorisation + the R1.4 sector projectors + the global $\mathbb{Z}_6$ centre (a68ee92a75be) + the $\mathbb{Z}_2$ orbifold on $S^1_{\mathbb{Z}}$ (ac4d2df3e708), and confirm $\Pi_i \Pi_j = \delta_{ij} \Pi_i$ and $\Pi_q \Pi_\ell = 0$ to machine precision **independently of the frozen labeling**; re-hash and confirm 3b8d68559f5e regenerates from the canonical description. This is a bounded linear-algebra-over-representations task. Success $\rightarrow$ R4 moves from AUDIT to VERIFIED, and Lemma 3's premise becomes machine-real, not assumed. (Fallback if deferred: name AXIOM-SECTOR-ORTHOGONALITY — but reconstruction is *better* than axiomatizing, because orthogonality of group-theoretically defined projectors is a computation, not a brute fact.)
- **R5 — operator ledger / coverage harness regeneration.** Mount the proton-operator ledger CSV (bc1e4e84840a); regenerate the L.2 ledger target-blind and confirm byte-equality; attach an explicit mechanism witness to every L.2b "Covered" row (sector-crossing $\rightarrow$ identity / coloured-triplet $\rightarrow$ Ingredient 1 / KK $\rightarrow$ the §4.1 no-joint-quantum-number lemma); re-hash to a5b1e6f9d951 . Fail-closed: if any ledger row cannot be regenerated, or any L.2b mechanism cannot be witnessed, Gate 10 downgrades. This is the highest value-per-effort closeable item (execution + verification, no new physics) and would convert the "Claimed certificate pass" status into machine-real. **It would also be the natural place to ship the two editorial fixes (§4.3) so the regenerated ledger is internally consistent.**

Both are **owner/machine-lane tasks, not axioms**. Neither promotes the gate; both make the hand-proof's premises reproducible.

4.3 The two corpus editorial fixes

Recorded for the corpus owner (READ-ONLY here; PROMOTIONS:o). Neither opens an escapee; both are internal-consistency / mechanism-attribution fixes.

1. **DEFECT 1 — "QQQL HH (dim-7)" $\rightarrow$ single-H d=7 (or move to d=8).** In the Appendix L ledger (o2 line 224), relabel the row from the double-Higgs form "QQQL HH (dim-7)" to the single-field form "QQQL·H (dim-7)" so it matches L.2b (o2 line 318: "QQQLΦ") and the CR10 companion (o3 line 198: "QQQL·Φ"). If the double-H form is intended for completeness, move it to a clearly-labeled d=8 row. Rationale: a d=6 core dressed by two scalars is mass dimension 8, not 7; the manuscript is currently internally inconsistent on this one row. Both forms retain the quark–lepton crossing, so the conclusion is unaffected.
2. **DEFECT 2 — footnote the $\bar{d}^c \bar{d}^c \bar{u}^c$ row.** In the Appendix L ledger (o2 line 227), footnote the $\bar{d}^c \bar{d}^c \bar{u}^c$ (*n*-side, quark-only $\Delta B=1$) row that **three Weyl fermions are not a Lorentz scalar**, so the operator closes only at higher dimension, and that it is killed by **Ingredient 1 (absent coloured-triplet mediator)**, not by the projector identity $\Pi_q M \Pi_\ell = 0$. Rationale: presenting it as a clean stand-alone operator misrepresents which mechanism kills it (this is exactly the quark-only escape route the census disposes of via Ingredient 1).

4.4 Hardening roll-up

Hardening move	Targets	Technique	Realistic endpoint	Promotes gate?
13D KK-overlap all-order analysis	STEP4 KK no-go (premise) + R1-all-order KK channel	Frobenius triality selection rule on already-computed $K_{\alpha}/S^2/S^1$ spectra	STEP4 claim $\rightarrow$ discharged lemma, all-order in KK number; R1-all-order KK channel closed; higher-dimension local-operator channel stays OPEN	No (R1-all-order as a whole stays OPEN; gate stays OPEN)
Bundle-orthogonality reconstruction (R4)	Lemma 3 premise	owner artifact + machine-lane	AUDIT $\rightarrow$ VERIFIED (premise machine-real)	No
Ledger / coverage harness regeneration (R5)	"Claimed certificate pass" reproducibility	machine-lane, fail-closed	BLOCKED_INPUTS $\rightarrow$ VERIFIED	No
Editorial fix 1 (QQQL HH $\rightarrow$ single-H)	corpus internal consistency	owner edit	corpus-consistent ledger	No
Editorial fix 2 ($\tilde{d}^c \tilde{d}^c \tilde{u}^c$ footnote)	mechanism attribution	owner edit	mechanism correctly attributed	No

The honest expected outcome of this hardening campaign: STEP4 discharged into a reconstructed theorem (all-order on the KK channel), R4 and R5 moved from AUDIT to VERIFIED, the two editorial defects fixed — leaving the gate **OPEN but with the $d \leq 7$ axiom floor, one fewer un-discharged premise, machine-verified bundle orthogonality, and a regenerated ledger**. That is a real honesty/reproducibility gain. It is **NOT a promotion**: R1-all-order (the higher-dimension local-operator channel) stays OPEN, the lifetime stays Diagnostic, the Battle verdict stays SHARED-OPEN, M_R stays UNKNOWN, and so the gate stays OPEN. **Ceiling: serious candidate, NOT validated**.

Appendix — frozen-object hash index (load-bearing; READ-ONLY)

Carried verbatim from the dossier for self-containment. **Nothing here was mutated**.

Branch `dcc66f1b2685` · manifest meta `a5b1e6f9d951` · sector projectors $\Pi_u, \Pi_d, \Pi_e, \Pi_\nu$ `3b8d68559f5e` (R1.4) · FCNC / mediator no-go theorem (full) `fff4b433b7b3` (R1.6) · operator-class declaration `551488d06011` (R1.6) · Cartan-torus modulus $\tau = \omega$ `03b30a9c931a` · global $\mathbb{Z}_6$ centre `a68ee92a75be` · $\mathbb{Z}_2$ orbifold on S^1_Y `ac4d2df3e708` · hypercharge bundle (no coloured-Higgs partner) `44516f6400ae` · proton-operator ledger CSV `bc1e4e84840a` . **Macro-projectors $\Pi_q, \Pi_\ell, Q_{BRST}$, the no-mediator identity $\Pi_q M \Pi_\ell = 0$, empty-cohomology statement: derived / standard — no separate R1 hash. M_R (dim-5 Weinberg / seesaw Majorana scale): no hash — UNKNOWN/open (`B610-FROZEN-MR`)**. Machine certificate: `certificates/G10_proton_safety/` (25 randomized class members; X -like off-block $\rightarrow$ FAIL confirmed).

Already-computed pure-geometry KK + Dirac spectra used in §4 (target-blind; δ does not appear):
`.../rendered/TOE/SPECTRA_COMPUTATION_2026-06-24/spectrum_K6.md` + `.csv` (K6 scalar Laplacian + spin-C Dirac, triality-0 selection rule), `.../spectrum_S2.md` + `.csv` (S^2 spherical harmonics + Wu–Yang monopole Dirac, $q \in \{0,1,2\}$),
`.../spectrum_S1.md` + `.csv` ($S^1_Y/\mathbb{Z}_2$ KK tower + $\mathbb{Z}_2$ chiral projection).

Closing honest statement

SG-9 is **OPEN** — by the grading rubric, *any* open piece $\Rightarrow$ OPEN — and this closure result does not change that gate status. What it does change is the standing of the central residual **R1**: its **$d \leq 7$ slice is now Reduced to Axiom — AXIOM-PROTON-DECAY-OPERATOR-COMPLETENESS- $d \leq 7$ — UPHELD on an independent refute-before-upgrade audit**. A bounded operator census through mass dimension 7 empties the escapee bin and confirms the standard operator types; the census logic (Lemma 1 quark–lepton crossing, the complete Weinberg/Wilczek–Zee $d=6$ basis, the complete Lehman $d=7$ basis, escapee-bin-empty, KK-no-bypass) is mathematically correct through $d \leq 7$ and was independently re-derived and confirmed. The result survives the three kill-tests that mattered — no-target-loading, reduce-not-relabel, and the A5-actor demotion-does-not-apply (SG-9’s universal negative is **bounded** to $d \leq 7$, where a finite enumeration genuinely closes it, unlike A5’s unbounded competitor matrix). Two corpus-editorial defects were found (“QQQL HH (dim-7)” is actually $d=8$; $\tilde{d}^c \tilde{d}^c \tilde{u}^c$ is not a stand-alone Lorentz scalar) — both non-fatal, neither opens an escapee, both have a recommended fix.

The honest open surface is equally clear and **must travel with the label**: all-order ($d > 7$) physical completeness stays **OPEN/computation-debt**; the hand-proof rests on **two un-discharged premises** carried as explicit conditions (R4 bundle-orthogonality AUDIT; STEP4 KK-tower no-go = claim — §4.1 is the concrete attack on the

latter, all-order in the KK number, via the K_6 Frobenius triality selection rule on the already-computed spectra); dim-6 BV is **SHARED-OPEN** across all GUT-class geometries (no Fable win; a filter, not a determiner); the dim-5 Weinberg / seesaw scale M_R is **UNKNOWN** and kept separate from the $\Delta B=1$ claim; the numerical lifetime is **Diagnostic only**. The five binding integration conditions are honored verbatim; the disposition is **AXIOM-CLOSED by a bounded hand-proof, explicitly NOT a regenerated machine certificate**; and the gate roll-up does **not** upgrade off **OPEN** (the pre-integration audit's word "PARTIAL," presented as the strictly-more-conservative OPEN).

PROMOTIONS: o; frozen branch `dcc66f1b2685 / a5b1e6f9d951` **READ-ONLY; given-E $\neq$ derivation of E; a selection-rule pass on a declared class $\neq$ a proof of absolute proton safety; nothing applied, nothing deployed. Honest ceiling: serious candidate, NOT validated.**

Closure result assembled 2026-06-25 by merging the SG9_PROTON_SAFETY_HANDOFF specialist closure and the independent refute-before-upgrade audit. Companion to `01_DOSSIER_SG9_PROTON_SAFETY.md` (the attack dossier). Our geometry (frozen 13D K_6 branch) only.