SG-5 — Electroweak Embedding / EWSB (GUT Gate 5): CLOSURE RESULT — Fable_Version rendered package. Rendered from `SG5_ELECTROWEAK_CLOSURE_RESULT.md`; frozen technical content unchanged by rendering.

SG-5 — Electroweak Embedding / EWSB (GUT Gate 5): CLOSURE RESULT

> **Status — binding header. Gate-level roll-up: OPEN** (any open piece ⇒ OPEN, per the grading rubric). **Charge/embedding leg: AXIOM-CLOSED + DERIVED-GIVEN-E.** $Q=T_3+Y$ and $SU(2)_L\times U(1)_Y\to U(1)_{em}$ are DERIVED-GIVEN-E; the minimal-actor selector + $n_H=1$ + Z_6 table are AXIOM-CLOSED; v_{EW} is accepted as a measured-but-irreducible second dimensionful anchor (alongside M_{Pl}), realized — not derived — by the Wilson-line VEV. **Custodial $\rho_{tree}=1$: OPEN — BLOCKED / computation-debt** (the headline correction). The tree-level $\rho=1$ is only the textbook single-doublet rep-algebra tautology; it is **not** a geometric readout, because the electroweak gauge-boson mass matrix M^2_{WZ} is verifiably **absent** from the corpus. **Hierarchy / lightness of v: OPEN / RELOCATION** (SCALE firewall uncrossed — not a hierarchy derivation). **Open audits:** R5 η_{BK} provenance SUSPECT; R6 v/m_h reproducer AUDIT/BLOCKED; R8 $N_b=N_f$ BLOCKED. **PROMOTIONS:0.** Frozen branch `dcc66f1b2685` / `a5b1e6f9d951` READ-ONLY. Honest ceiling: serious candidate, NOT validated. **Provenance:** specialist result → refute-before-upgrade verification (`wh3rfp8ow`) → **Route B** blind mass-matrix computation (`wi00pp5t0` ; physics + disposition both SOUND/HONEST). Verified 2026-06-25.

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§1. What closes — the charge/embedding leg

Leg	Disposition	Basis
Charge $Q=T_3+Y$	**DERIVED-GIVEN-E**	Componentwise on the SM multiplets, given the frozen Z_6 parity table; the gate's strongest leg.
EWSB $SU(2)_L\times U(1)_Y\to U(1)_{em}$	**DERIVED-GIVEN-E**	The single Wilson-line/Hosotani doublet $H\sim(1,2,+½)$ with a neutral $SU(2)_L$-direction VEV (Q-preserving, $T_3=-Y$).
Minimal EW-breaking actor (uniqueness)	**AXIOM-CLOSED** (minimality selector)	Elimination inventory correct (singlet no-break; real triplet $\rho=2$; complex triplet $\rho=½$; higher fail/non-minimal). Uniqueness is **genuinely conditional on minimality** — exotic multiplets *can* give $\rho=1$, so minimality travels as the named axiom. **"Custodial" must be stripped from the axiom name** — bundling it RELABEL_FAILs (custodial ≈ "gives $\rho=1$"). `AXIOM-CUSTODIAL-FROM-DOUBLET` stays *named-but-not-banked*.
$n_H=1$ winding · Z_6 table	**AXIOM-CLOSED**	Minimal admissible winding; selected center-locking table (=Z_6 finest-ness OPEN).
v_{EW}	**measured-but-irreducible**	The second dimensionful anchor; realized by `v_EW = θ_H*/(2πR_γ)` , **not** derived.

§2. The headline correction — custodial ρ=1 is BLOCKED, not DERIVED

Route B (target-blind) result: ρ=1 = OPEN / BLOCKED-computation-debt. The tree algebra `M_W²=g²v²/4, M_Z²=(g²+g'²)v²/4, M_γ=0 ⇒ ρ_tree=1` is correct **only as SM rep-algebra on the doublet label**. It is **not** a readout from the frozen geometry, because:

- **The EW gauge-boson mass matrix M²_WZ is never built on the frozen branch.** The corpus has only coupling-level normalization (`1/g_em² = 1/g₁² + 1/g₂²` ; `e = gg'/√(g²+g'²)`) and `M_W ← v` via Fermi — it **never** computes M_Z from `|D_μH|²`, never the (W³,B) block, never ρ. The SM θ_W mixing is **written by hand**, not derived. (The dossier's W11 records the custodial object as "DOES NOT EXIST.")
- **Three un-discharged premises** block DERIVED-GIVEN-E: (1) no certificate the protected zero-mode spectrum carries **no second VEV source** (triplet/extra doublet); (2) the **S²-vs-S¹_Y kinetic co-normalization** of the off-diagonal `-gg'v²/4` term is never formed (the two gauge factors live on geometrically distinct sub-manifolds); (3) no **EW KK gauge-boson mixing** (the gauge-Higgs-unification T-parameter) computation.

Each is a real geometric channel that could break custodial SU(2). So **neither ρ=1 (case a) nor ρ≠1 (case b) is earned** — custodial breaking is *possible-but-undemonstrated*. Naming ρ=1 a closure would be target-loading.

To unblock (the concrete computation-debt)

Freeze (i) the SU(2)_L reduction integral over S² and the U(1)_Y reduction over S¹_Y/$\mathbb{Z}_2$ in a **common normalization**; (ii) `|D_μ⟨H⟩|²` assembled into `{W±; W³,B}` with the common `v=θ_H*/(2πR_γ)` ; (iii) a zero-mode-spectrum certificate (no 2nd VEV source); (iv) the EW KK mixing correction to M_Z/M_W. Then ρ reads off **blind** — and either earns ρ=1 = DERIVED-GIVEN-E or finds it breaks vs the measured ρ₀=1.00038±0.00020 (a live falsifier). Until then: **do not bank ρ=1 as closure.**

§3. What stays OPEN (gate roll-up)

- **R1/R2 — hierarchy / lightness of v:** OPEN / RELOCATION. θ_H *is read from the one-loop minimum (~85% of the hierarchy undelivered); the SCALE firewall returned RELOCATION (μ_cell has no v-independent readout) and that verdict is carried, not overturned. v_EW is a genuine second anchor; its lightness is undelivered*.
- **R3 — custodial ρ=1:** OPEN (BLOCKED/computation-debt, §2).
- **R5 — η_BK provenance:** SUSPECT. **R6 — v/m_h reproducer:** AUDIT/BLOCKED. **R8 — N_b=N_f:** BLOCKED_INPUTS.

The re-scoping (close SG-5 as the EW embedding/actor gate, v_EW a transparent measured anchor, hierarchy out) is **HONEST** — it buries no 246 GeV and keeps the hierarchy loudly OPEN. But the **gate is governed by its least-closed residual**, so it stays **OPEN**. The charge/embedding leg is the closest-to-closed; ρ=1 is the weakest (BLOCKED).

§4. Honest one-line

> SG-5's **charge/embedding leg is AXIOM-CLOSED + DERIVED-GIVEN-E** (Q=T3+Y, EWSB to U(1)_em, minimal Wilson-line doublet, v_EW a measured anchor). **Custodial ρ=1 is OPEN/BLOCKED** — the geometry never builds the W/Z mass matrix, so tree ρ=1 is a rep-algebra tautology, not a derivation. The **hierarchy is OPEN/RELOCATION**, and the **gate is OPEN**. NOT a hierarchy derivation; NOT a proof that the geometry yields ρ=1. Serious candidate, NOT validated.

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§5. The completion endpoint — SG-5A / SG-5B split (formalized 2026-06-25)

A specialist "completion-stack" pass froze SG-5's **strongest honest endpoint**, and an independent refute-before-upgrade audit (`wmtzxcmts`) found it **PROMOTION-SAFE** (all five kill-tests pass; it closes nothing new — it *confirms* the disposition above). The endpoint formalizes SG-5 as two sub-objects, with the **gate itself OPEN**:

- **SG-5A — Electroweak embedding / actor / mass matrix.** Status: **OPEN**, with one open computation-debt leg. *Closed legs:* Q=T3+Y, EWSB→U(1)_em (DERIVED-GIVEN-E); minimal actor / n_H=1 / Z6 (AXIOM-CLOSED); v_EW (measured-but-irreducible anchor). *Open leg (R3):* the geometric W/Z mass matrix + no-second-VEV + KK Schur correction — i.e. ρ=1 must still be **earned**, not asserted. (Avoid "near-terminal" — it is OPEN with one named computation-debt leg.)
- **SG-5B — Electroweak hierarchy / θ_H* / v-lightness.** Status: **OPEN / RELOCATION**, **exported** to the tracked `SG5B_EW_HIERARCHY_EXPORT_LEDGER.md` (SCALE / SG-6). v_EW is *not* derived inside SG-5A; the lightness of v is a separate transmutation problem.

The R3 upgrade path (frozen recipe to earn ρ=1 = DERIVED-GIVEN-E). Build, target-blind: (i) the SU(2)_L (S^2) + U(1)_Y (S^1_Y/$\mathbb{Z}_2$) reduction integrals in **one common normalization**; (ii) the assembled $\int|D_\mu\langle H\rangle|^2 \to \{W^\pm; W^3, B\}$ mass matrix with the common v; (iii) a **no-second-VEV** zero-mode certificate; (iv) the EW KK **Schur-complement** correction $M_{eff}^2 = M_{00}^2 - M_{0n}^2 (M_{nn}^2)^{-1} M_{n0}^2$ (or a bound). Then ρ reads off blind: ρ=1 → DERIVED-GIVEN-E · ρ≠1 → FINDING (vs ρ0=1.00038±0.00020) · data absent → still BLOCKED.

Audit flags — resolved 2026-06-25: (a) the SG-5B export now has a **tracked destination ledger**, `SG5B_EW_HIERARCHY_EXPORT_LEDGER.md` (status OPEN/RELOCATION) — tracked, not merely asserted; (b) the completion-stack artifacts are now **archived + re-run-verified** (`SG5_complete_endpoint_stack/` , 23 files, manifest **22/22 intact**): `validate_endpoint.py` → **PASS (exit 0, fail-closed)**; `compute_textbook_rho.py` → ρ=1 (rep-algebra only, correctly labeled); `reproduce_eta_BK.py` → η_BK = 0.009721281516312024 (exact, 0 error; provenance still BLOCKED_AUDIT). The validator's PASS is now a **re-run certificate**, not self-reported — and it passes *because nothing is promoted* (`verdict.json` : promotions:0, target_blind:true, original_SG5_rollup:OPEN, all R3/R5/R6/R8 evidence flags false).

Net: SG-5 stays **OPEN**. The endpoint is the cleanest frozen disposition **plus** the precise recipe to close ρ=1 — it does **not** close it. PROMOTIONS:0.

§6. Completion routes + axiom reduction (all-paths stack — re-verified 2026-06-25)

A follow-on specialist pass produced `SG5_all_paths_to_completion_stack/` (archived in the corpus; **re-run-verified** by direct examination: `validate_all_paths.py` → **PASS** (fail-closed; promotions:0; status OPEN); `reproduce_eta_BK.py` → η_BK = 0.009721281516312023; missing certificates honestly marked `.MISSING.md`). It lays out **all four completion routes** + the **axiom floor** and — like the endpoint — **closes nothing new** (R3 BLOCKED, gate OPEN).

The four routes (pick per residual, by the evidence): - **Path A — DERIVED-GIVEN-E:** supply the R3 certificates (common-normalization integrals · no-second-VEV · KK-Schur bound) → read ρ blind → DERIVED-GIVEN-E iff the geometry yields C_W=C_3=C_Y=C_3Y=1, Δ_KK=0/bounded. - **Path B — Reduced to Axiom: only for walls, NOT for missing computations** (explicit guard against relabeling computation-debt). - **Path C — FINDING:** record ρ≠1 / custodial breaking as a falsifiable finding. - **Path D — BLOCKED:** name the exact missing owner artifacts (the current state of R3/R6/R8).

Axiom-reduction ledger (banked vs candidate): - **BANKED:** `AXIOM-FROZEN-ADMISSIBILITY-GRAMMAR` (governing rule) · `AXIOM-MINIMAL-EW-BREAKING-ACTOR` (AXIOM-CLOSED, stripped of any custodial/ρ claim) · `AXIOM-EW-SCALE-ANCHOR-SEPARATION` (export discipline) · `AXIOM-BLIND-STRUCTURAL-PROVENANCE` (method axiom). - **CANDIDATE / computation-debt (NOT banked):** `AXIOM-SINGLE-EW-VEV-SOURCE`, `AXIOM-COMMON-EW-KINETIC-DESCENT`, `AXIOM-EW-KK-SCHUR-DECOUPLING` — each terminalizes **only** when its R3 certificate is supplied (Path A), never by being named. **`AXIOM-CUSTODIAL-FROM-DOUBLET` is explicitly NOT banked** (it would relabel ρ=1). - **Three-root compression (conditional — only *if* the certificates succeed):** frozen admissibility · minimal single EW actor · metric-consistent descent with KK control.

The discipline holds throughout: genuine structural axioms banked; the R3 pieces kept as **computation-debt** (candidate axioms, not relabeled closures); the hierarchy exported; ρ=1 BLOCKED until *earned* by the geometry. **Net unchanged: SG-5 OPEN. PROMOTIONS:0.**

§7. Path A executed — BLOCKED (conditional-DERIVED); the co-normalization blocker dissolved *for ρ* (2026-06-25)

Path A (assemble the geometric W/Z mass matrix from the frozen geometry, read ρ blind) was executed and adversarially verified (workflow `wex5xj2ae`; physics + disposition passes both **HONEST**; target-blind; PROMOTIONS:0). Outcome: **BLOCKED (conditional-DERIVED)** — *not* a legitimate upgrade to DERIVED-GIVEN-E — but with a real banked advance and a sharply-narrowed remaining blocker.

Banked advances (UPHELD — genuine, not promotions): 1. **The co-normalization blocker is *void for ρ*.** With independent overlaps a_W (S^2), a_B (S^1_Y), the assembled matrix gives **ρ_tree = C_W/C_3 exactly** — C_Y and C_3Y cancel, so the S^2-vs-S^1_Y co-normalization **a_B drops out of ρ entirely** (explicit algebra, not assertion). The thing three prior passes were stuck on (blocker #2) is a **red herring for ρ**. (It is still owed for the *absolute* masses M_W, M_Z.) 2. **The prior pass-condition was**

over-strong: ρ=1 needs only **C_W=C_3 and C_3Y²=C_3·C_Y**, not all four C's=1. 3. **Photon-masslessness (det of the neutral block = 0): DERIVED-GIVEN-E**, normalization-independent — forced by the Q=T₃+Y-preserving VEV (U(1)_em unbroken).

Why it stays BLOCKED (the adversary's catch): the headline "C_W=C_3 because one su(2) field / one S² wavefunction" is an identity of an *assumed fundamental-doublet* covariant derivative. The frozen object is a **non-custodial Hosotani** Higgs (A_y = θ_H/(2πR_γ)·T_H, T_H∈su(2)_L, the doublet tensored in by hand, **SO(4) eliminated**), so the W/Z mass term descends from the **commutator** `Tr|[A_μ, A_y]|²` — and C_W=C_3 is **not** read out of the frozen data: with no custodial symmetry to protect it, it must be earned from the VEV-weighted commutator overlaps `Tr|[T^a, T_H]|²` (a=1,2,3 weighting equally).

ρ_tree=1 is therefore CONDITIONAL-DERIVED on three un-discharged premises: (i) **C_W=C_3** via the Hosotani commutator overlaps — a **newly-named owner-artifact** `R3_hosotani_commutator_overlap` (the prior passes mis-cited the co-normalization; *this* is the real load-bearing blocker); (ii) **no-second-VEV** (R8-gated, needs the m₀(p,q) zero-weight table); (iii) **KK-Schur ≤ 10⁻⁴** (dimensional hint ~10⁻²⁹; the l=0→1 / n=0→1 overlap not computed).

R3: BLOCKED. GATE: SG-5 OPEN (unchanged). PROMOTIONS:0. The net is precision, not closure: blocker #2 dissolved for ρ, the det=0 leg DERIVED-GIVEN-E, and the remaining blocker sharpened from "the whole co-normalization" to one specific commutator-overlap certificate. ρ₀=1.00038±0.00020 used as a post-hoc falsifier only — input to nothing.

§8. The commutator overlap COMPUTED — ρ is physical but UNDETERMINED (BLOCKED, sharper); 2026-06-25

§7's load-bearing artifact `R3_hosotani_commutator_overlap` has now been **computed** and adversarially verified (two independent routes + an independent re-derivation; reproducer `SG5_R3_RHO_REPRODUCER.py`, all checks PASS to machine precision). The result sharpens the blocker one level deeper and is the **resolved** R3 disposition.

What was computed. The VEV-weighted commutator overlaps are, exactly,

$$O_a = \left\langle \mathrm{Tr}\,|[T^a, T_H]|^2 \right\rangle = \tfrac{1}{2}\left(|h|^2 - h_a^2\right), \qquad T_H = \textstyle\sum_b h_b T_b,$$

so the tree W/Z mass matrix gives

$$\rho_{\mathrm{tree}} = \frac{O_1 + O_2}{2\,O_3} = \frac{1}{2} + \frac{h_3^2}{h_1^2 + h_2^2} \;\in\; \left[\tfrac{1}{2}, \infty\right), \qquad \rho = 1 \iff 2h_3^2 = h_1^2 + h_2^2 \;(\text{a } 54.74^\circ \text{ cone}).$$

The decisive new fact — V_Hos is exactly FLAT in the Wilson-line direction. The doublet holonomy eigenvalues are $\pm\theta_H/2$ **independent of the direction** of T_H (the KK weight-multiset is rotation-invariant), so $V_{\mathrm{Hos}}(\theta_H, \text{direction}) = V_{\mathrm{Hos}}(\theta_H)$: the one-loop potential **selects no direction.** ρ=1 (the cone) is one ordinary, **non-preferred** level set, not a minimum.

ρ is physical, but its value is undetermined. The EW-neutral 3-axis (T₃) is **pinned** by the frozen Z₆ fermion-charge table (Q=T₃+Y must reproduce the observed charges), so ρ is a genuine gauge-invariant

observable (invariant under the residual Q-preserving $U(1)_3$) — **not** a gauge artifact. But because V_{Hos} is flat in the tilt of T_H relative to that pinned axis, the value of h_3 — hence ρ — is a **flat modulus**, undetermined over $[\frac{1}{2}, \infty)$. The custodial SO(4)/SO(5) coset that would force the cone was **eliminated**; there is **no tree-level su(2)_L-anisotropic term** in the frozen data to lift the flat direction.

Two routes, one missing datum. Route A (numerical V_Hos) → BLOCKED (tilt physical, value flat). Route B (structural) → leans ρ=1 by the lone-doublet accidental custodial, but only **conditional** on two embedding facts (P1∧P2) *not present in the frozen corpus*. They agree on every computed fact and disagree only on disposition, and the disagreement reduces to a single owner-artifact: **whether the frozen γ-cycle / Z_6 embedding pins the EW-neutral 3-axis independently of T_H.** Per the reconciliation rule (material disagreement → BLOCKED) and the Prime Directive (**ANCHORED ≠ DERIVED**; a conditional gauge-equivalence lean is not a derivation), the disposition is **BLOCKED**.

Status — resolved: R3 BLOCKED (sharper). ρ=1 is neither derived nor falsified — it is UNDETERMINED. The frozen geometry, as given, **does not fix ρ.** Closing it requires one of: the γ-cycle/Z_6 orthogonality certificate (does the 3-axis pin independently of T_H?), or the two-loop Yukawa/g′-corrected V_{Hos} (where $y_t \neq y_b$ and $g' \neq 0$ first break the adjoint isotropy). **GATE: SG-5 OPEN. PROMOTIONS:0.** We make **no** claim the model predicts ρ=1, and introduced **no** parameter tuned to the measured value; $\rho_0 = 1.00038 \pm 0.00020$ was an input to nothing.