SG-4 — Hypercharge / Anomaly (GUT Gate 4): CLOSURE RESULT — Fable_Version rendered package. Rendered from `SG4_ANOMALY_CLOSURE_RESULT.md`; frozen technical content unchanged by rendering.

SG-4 — Hypercharge / Anomaly (GUT Gate 4): CLOSURE RESULT

> **Status — binding header. Gate-level roll-up: OPEN** (any open piece ⇒ OPEN, per the grading rubric). **Local leg — Global Gauge Admissibility: AXIOM-CLOSED + DERIVED-GIVEN-E.** The local gauge-representation admissibility of the upstream-generated spectrum E is closed: `O_SG4_local(E_frozen) = 0`. **Open residuals keeping the gate OPEN:** Z6-finest-ness (declared, not forced), R4 mixed anomaly ξ_R4 (OPEN/computation-debt), BV-BRST descent + descent measure (UQF-4/UQF-7, NO-KNOWN-ROUTE), 16/16 quantum coverage (OPEN, ~10/16 at cert grade), machine certs G03/G05 (AUDIT/BLOCKED). **NOT** a derivation of E; **NOT** "anomaly cancellation selects the SM" (that is DISSOLVED/false). **PROMOTIONS:0.** Frozen branch `dcc66f1b2685` / `a5b1e6f9d951` READ-ONLY. Honest ceiling: serious candidate, NOT validated. **Provenance:** specialist result + independent refute-before-upgrade verification (workflow `wtiihur36`; physics + disposition passes both HOLD/HONEST; break-test: no arithmetic error). Verified 2026-06-25.

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§1. The reframing — Global Gauge Admissibility (adopted)

SG-4 is **not "the hypercharge table."** It is the **vanishing of an obstruction map** on the chiral spectrum E generated upstream (SG-2 group, SG-3 spectrum):

$$O_{\mathrm{SG4}}(E) = \big(O_{\mathrm{descent}}(E),\ O_{\mathrm{charge}}(E),\ O_{\mathrm{anomaly}}(E),\ O_{\mathrm{Witten}}(E)\big), \qquad O_{\mathrm{SG4}}(E_{\mathrm{frozen}}) = 0.$$

The SM hypercharge table is the **coordinate shadow of bundle descent**, not a primitive. This repackaging adds no claim and changes no status — it states the local leg at the invariant level. The headline "**AXIOM-CLOSED + DERIVED-GIVEN-E**" is valid for this **local leg only**, never as a gate label.

§2. Per-leg disposition (verified)

Leg	Disposition	Basis
Six local anomaly ledgers ([U(1)_Y]³, [grav]²U(1)_Y, [SU(2)]²U(1)_Y, [SU(3)]²U(1)_Y, [SU(3)]³, Witten mod-2)	**DERIVED-GIVEN-E**	All re-derived to exact zero by rational arithmetic on the 1-gen spectrum; $\sum Y^2=10/3\neq 0$ ⇒ cancellation **specific, not trivial**. A check that E passes, **not** a derivation of E.
Charge table $Q=T_3+Y$	**DERIVED-GIVEN-E**	ν neutral, d at −1/3, no per-multiplet fit.
Hypercharge lattice $Y\in(1/6)\mathbb{Z}$	**DERIVED-GIVEN AXIOM-Z6-DECLARED**	Center-locking `ω₃^{k₃}ω₂^{k₂}ω₆^{6Y}=1` ⇒ $6Y\in\mathbb{Z}$; verified all 6 multiplets.
Z_6 center quotient	**AXIOM-CLOSED (declared)**	Selected, frozen.
Z_6-finest-ness ($\Gamma=Z_6$ vs $\Gamma\leq Z_6$)	**OPEN**	AX-FINEST rejected on no-target-loading; line-operators pin only $\Gamma\leq Z_6$ (q|6, not q=6).
"Anomaly cancellation selects the SM"	**DISSOLVED / false**	A filter, not a determiner — any $R\oplus\bar{R}$ cancels ⇒ infinitely many anomaly-free spectra; `E_frozen ∈ ker O_SG4`, **not** `ker O_SG4 = {E_SM}`.
R4 mixed 't Hooft anomaly ξ_R4	**OPEN / computation-debt**	STILL_SUBTLE; operative $d_3=Q_1=\beta P^1$; untwisted $\mathbb{Z}_3$ survives; $\tau\_K_6=(2,2)$ twist-correction the sole lever; **no value, no hash**; NOT-A-WALL (inert for the mass gap).
BV-BRST descent (UQF-4) + descent measure (UQF-7)	**AUDIT / NO-KNOWN-ROUTE**	AXIOM-CLOSED only as an inherited-standard-QFT stance; not proven. Genuinely tracked in `OPEN_WALLS_REGISTER.md`.
Machine certs G03/G05	**AUDIT / BLOCKED**	Not re-run; hand-reproducible mitigant.
Full 16/16 quantum-consistency coverage	**OPEN**	~10/16 at certificate grade.
Whole SG-4 / Gate 4 roll-up	**OPEN**	Least-closed residual governs: Z_6-finest + ξ_R4 + BV-BRST + 16/16 + certs all open.

§3. The six ledgers (witnesses)

1. `[U(1)_Y]³ = ΣY³` : per-field `36·mult·Y³ = {1, −32, 4, −9, 36}` → **0**.
2. `[grav]²U(1)_Y = ΣY` : `{+1, −2, +1, −1, +1}` → **0**.
3. `[SU(2)]²U(1)_Y` : `3·(1/6) − 1/2 = 0`.
4. `[SU(3)]²U(1)_Y` : `2·(1/6) − 2/3 + 1/3 = 0`.
5. `[SU(3)]³` : colour vector-like ($Q_L + u^c + d^c \Rightarrow 1-1$) → **0**.
6. Witten `[SU(2)]` mod-2: #doublets `= 3+1 = 4`, even → **no anomaly**.

Diagnostic: `ΣY² = 10/3 ≠ 0` — the cancellation is **specific**, not an automatic consequence of a trivial sum.

§4. The factorization (why the re-scoping is honest, not a dodge)

`O_SG4_local(E) ⊂ O_full_quantum(E)`, where `O_full_quantum = (O_SG4_local, O_BV-BRST, ξ_R4, …)`. SG-4 owns the **local** obstruction; BV-BRST descent + R4 are **higher quantum-completion obstructions** that genuinely **exist as tracked open walls** (UQF-4, UQF-7 under GENUINELY-OPEN-NO-ROUTE; R4 Tier-C, value UNKNOWN). They are kept **loudly open with named obstructions**, not buried inside SG-4 — that is what makes the factorization legitimate rather than a goalpost-move. The gate stays **OPEN** until they close.

§5. Honest one-line

> SG-4's **local gauge-representation admissibility** is closed: `O_SG4_local(E_frozen)=0` (six anomaly ledgers DERIVED-GIVEN-E, $Q=T_3+Y$ + $Y\in(1/6)\mathbb{Z}$ DERIVED-GIVEN-E given the AXIOM-CLOSED Z_6 table). The **gate is OPEN** on Z_6-finest-ness, ξ_R4, BV-BRST descent, 16/16 coverage, and the machine certs. SG-4 does **not** derive E and does **not** show anomaly cancellation selects the SM (DISSOLVED/false). Serious candidate, NOT validated.