Muon Decay as a Weak-Force Check

How a shape built to explain particles also makes the muon decay
by exactly the textbook weak interaction — and how you can check it

A mostly non-technical note for anyone who loves quantum physics
— written to be checked by specialists, and read, equations and all, by everyone else —

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Before we start

This note checks one small but unforgiving thing. If the thirteen-dimensional shape built to explain nature's particles really contains the Standard Model's electroweak sector, then it must reproduce the simplest quantum weak process: a muon decaying into an electron and two neutrinos. We compute that decay the standard way, then show the same interaction arising from the thirteen-dimensional route, and place the two answers side by side. The aim is not to prove the full geometry — it is to show that the construction lands on the correct low-energy weak theory where it must.

Two honest words first. This is a **consistency check**, not a proof of the whole construction; and the shape itself is **not derived here**. You can read every word skipping every equation; the math is set off for anyone who wants to check it.

1. Where this check fits, and why muon decay

This note is the third in a short series of consistency checks on one thirteen-dimensional geometry. The companions check its gravity (*The Sun as a Lens*) and its electromagnetism (*Hydrogen as a Quantum Electrostatic Check*); this one checks its weak force. All three run the same way:

gravity: 13D geometry → 4D Einstein gravity → the bending of starlight

electromagnetism: 13D geometry → 4D QED → the Coulomb force that binds hydrogen

weak force: 13D geometry → 4D electroweak theory → the muon's decay

Muon decay is the cleanest weak test for one reason above all: it is **purely leptonic**. A muon, an electron, and two neutrinos — no quarks, no atomic nuclei, no strong-force complications. It runs entirely on the charged weak current, it tests the weak force's defining feature (its handedness — only left-handed particles take part), and at the low energies of the decay it collapses to a single contact interaction simple enough to integrate by hand. The familiar nuclear weak process, neutron beta decay ($n \to p + e^- + \bar{\nu}_e$), is the same interaction one level up, but it drags in the messy internal structure of protons and neutrons, so it makes a poor first test. We do muon decay first, then translate to beta decay near the end.

2. Route one: the standard weak theory

In the Standard Model, the left-handed leptons sit in weak doublets — (ν_e, e) and (ν_μ, μ) — and the charged weak bosons W^+ and W^- turn one member of a doublet into the other. That is what the weak force is: the muon emits a virtual W^- and becomes its neutrino; the W^- then turns into an electron and an antineutrino.

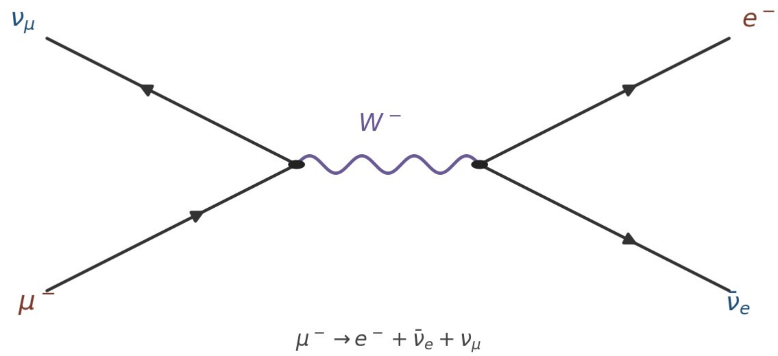

Figure 1. *Muon decay, the standard route. The muon converts to ν_μ by emitting a virtual W^-, which becomes e^- and $\bar{\nu}_e$. At energies far below the W mass, this W exchange shrinks to a single contact interaction.*

The interaction carries the weak force's quantum signature, the 'V−A' structure $\gamma^u(1-\gamma^5)$: only left-handed particles participate. The W is very heavy, so at the muon's low energy the virtual W cannot travel any distance; its exchange shrinks to a single point — a four-particle contact interaction. This is Fermi's interaction, with its strength set by

$$G_F/\sqrt{2} = g^2 / (8\, m_W^2),$$

relating the low-energy Fermi constant G_F to the weak coupling g and the W mass. Integrating over the ways the three outgoing particles can share the released energy gives the muon's decay rate — the inverse of its lifetime:

$$\Gamma_\mu = G_F^2\, m_\mu^5 / (192\pi^3),$$

ignoring the tiny electron mass and small radiative corrections. This is the measured answer (the muon lives about 2.2 microseconds). Call it route one.

3. Route two: the same decay from the geometry

Now the same decay, this time from the thirteen-dimensional shape rather than assumed.

The shape's internal space carries the electroweak forces as its low-energy vibrations: the $SU(2)_L$ gauge fields (three of them, W^1, W^2, W^3) and the $U(1)_Y$ field B. Two of the $SU(2)_L$ fields combine into the charged W^+ and W^-. The mathematics that makes the $SU(2)_L$ fields act on the lepton doublets is exactly the operation 'turn one member of the doublet into the other' — so once the shape supplies the doublets and the gauge coupling, the W-boson charged current isn't an extra assumption; it is forced.

The shape must also break the electroweak symmetry — the same Higgs mechanism (here, a Wilson-line winding) that gives the W its mass in ordinary physics — so the W comes out heavy, m_W = ½ gv. With a heavy W and the charged current in hand, the rest is identical to route one: at low energy the W exchange shrinks to Fermi's contact interaction, with the same relation G_F/√2 = g²/(8 m_W²), and the same decay rate follows.

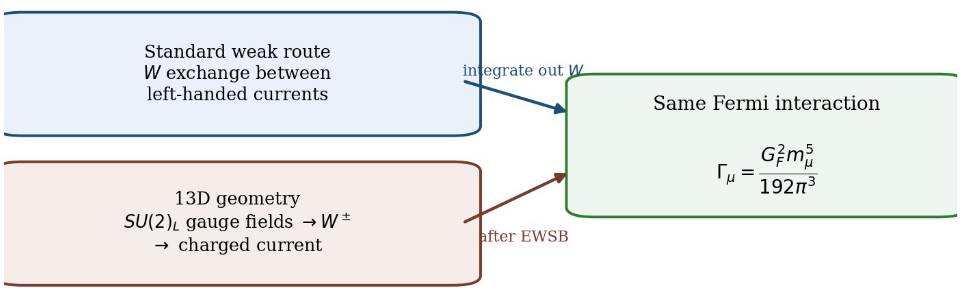

Figure 2. *Two routes, one weak interaction. The standard route assumes the W and its chiral coupling; the 13-dimensional reduction supplies them — SU(2)_L gauge fields and chiral doublets — and after electroweak breaking both collapse, at low energy, to the same Fermi interaction and the same decay rate.*

And, as in the two companion checks, this isn't circular: the shape was selected by requirements about particles that never mentioned muon decay. That the same shape reproduces the exact chiral, W-mediated interaction governing the muon's decay is a construction built for entirely different reasons landing on the right physics.

The full reduction — the gauge action, the chiral doublets, the charged current, the W mass, and integrating out the W — is in Appendix A, with what the corpus establishes directly and what is completed here spelled out there.

4. The two routes meet — and not just the muon

Step by step, the two routes line up on every ingredient of the weak interaction:

Quantity	Standard weak theory	13D reduction route
Gauge group	SU(2)_L × U(1)_Y	internal-geometry zero modes reduce to the electroweak gauge fields
Leptons	left-handed doublets	chiral lepton doublets from the geometry
Weak bosons	W^+, W^-	charged combinations of

Quantity	Standard weak theory	13D reduction route
		the SU(2)_L gauge modes
Chirality	V−A, left-handed	no-mirror / chiral projection
W mass	Higgs mechanism, m_W = gv/2	Wilson-line Higgs gives the same low-energy mass
Low-energy limit	integrate out W	integrate out the same W after reduction
Effective theory	Fermi interaction	*same*
Example	muon decay rate	*same*

Once the effective interaction is the same, the leading decay rate is the same. And muon decay is only the cleanest case. The same reduction supplies the whole electroweak sector at once, so every charged-current weak process comes along — every particle decay and weak scattering governed by W exchange. Muon decay is the front door; behind it is the ordinary low-energy weak interaction in general.

5. The nuclear bridge: beta decay

The weak force most people meet first is radioactive beta decay — a neutron turning into a proton, $n \to p + e^- + \bar{\nu}_e$. It is the same charged current, one level down. At the level of quarks, a down quark inside the neutron converts to an up quark (turning udd into uud), emitting the same W^- that becomes the electron and antineutrino:

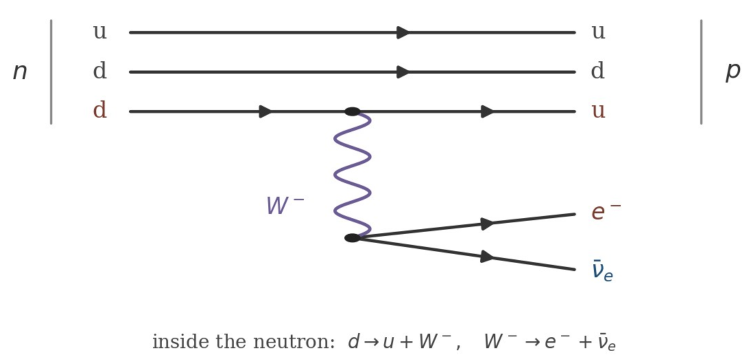

Figure 3. *The familiar nuclear weak force is the same charged current, applied to a quark inside the neutron: $d \to u + W^-$, then $W^- \to e^- + \bar{\nu}_e$. The spectator quarks carry the neutron (udd) into the proton (uud).*

So beta decay and muon decay are the same weak interaction, applied to quarks instead of leptons. The honest difference: the proton and neutron are strong-force bound states of quarks, so beta decay carries extra ‘form factors’ (such as g_A) that encode that internal structure — which is precisely why it is a poorer first test than the clean, purely leptonic muon. The weak vertex is identical; the nuclear packaging is what muddies it.

6. What this does, and does not, show

To keep the claim the right size: reproducing the charged-current weak interaction and the muon decay law is something *any* candidate theory has to do. Passing means the construction cleared a hurdle it genuinely could have failed — the weak force might have come out non-chiral, the W massless, or the current the wrong structure. But clearing it does not prove the larger construction.

What it does not show, plainly: it does not prove the full thirteen-dimensional construction; it does not compute the electroweak loop corrections; it does not, by itself, predict the numerical muon lifetime from nothing — that requires the weak coupling g and the W mass, which the construction routes but which rest on a separately computed electroweak scale; it does not remove the hadronic form factors from beta decay; and it does not solve neutrino masses or mixing. It is a necessary low-energy consistency check, and only that.

The real evidence for the construction lives where it matches numbers it was never built to match — the structure of the forces, the three families of matter, the particle properties that follow from a handful of measured inputs. This is one more door that opens.

7. The seam, now closed

As with the two companion notes, the step from ‘the geometry contains the electroweak sector’ to ‘therefore the muon decays correctly’ is written out in full in Appendix A. In one breath: the shape’s $SU(2)_L$ doublets and gauge coupling force the W charged current; electroweak breaking makes the W heavy; integrating it out at low energy gives Fermi’s interaction with $G_F/\sqrt{2} = g^2/8m_W^2$; and the three-body phase-space integral gives $\Gamma_\mu = G_F^2 m_\mu^5/192\pi^3$.

This check is, of the three in the series, the one the corpus supports most directly. What the corpus establishes — at ‘interface’ confidence — is not only the electroweak gauge sector ($SU(2)_L$ from the internal geometry, $U(1)_Y$ hypercharge, chiral doublets with no mirror partners, and electroweak breaking giving a massive W and Z) but the Fermi reduction itself: it carries the relation $G_F/\sqrt{2} = g^2/8m_W^2$ as a derived identity — one that would break against measurement if the geometry were wrong. What this note completes is the standard final step: assembling the decay amplitude and integrating the three-body phase space to the muon decay rate. And what remains genuinely conditional is named, not hidden: the reduction must give exactly the chiral (left-handed-only) doublets with no mirror sector, the canonically normalized gauge fields, the correct W and Z masses, the G_F relation, and no extra light charged weak boson. If all of those hold, the muon decay law follows.

Appendix A — The full chain, from the 13-dimensional electroweak sector to the muon decay rate

This appendix writes out, step by step, the derivation summarized in Section 7. It is not required reading for the story; it is here so the claim can be checked. The goal is narrow: show that the thirteen-dimensional geometry's electroweak subsector reduces to the ordinary low-energy weak interaction — the W charged current, Fermi's theory, and the muon decay rate.

A.1 The electroweak sector from the 13-dimensional geometry

The thirteen-dimensional construction carries the Standard Model gauge fields as the low-energy zero modes of its internal geometry. For the weak-force check, isolate the electroweak sector: the SU(2)_L gauge fields W^1, W^2, W^3 and the U(1)_Y field B, coupled to the left-handed lepton doublets,

$$L_e = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \qquad L_\mu = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \qquad \ell = e, \mu, \tau$$

through the electroweak covariant derivative:

$$D_\mu = \partial_\mu + ig\,\frac{\tau^a}{2}\,W^a_\mu + ig'\,Y\,B_\mu$$

The SU(2)_L generators τ^1 and τ^2 mix the two members of each doublet — which is exactly what the charged weak force does: it turns one member of a doublet (a neutrino) into the other (its charged lepton), and back.

A.2 The charged current

The charged combinations of the SU(2)_L fields are the physical W bosons, and the doublet kinetic term contains, on expansion, the charged-current interaction:

$$W^\pm_\mu = \tfrac{1}{\sqrt{2}}\left(W^1_\mu \mp iW^2_\mu\right)$$

$$\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}}\left(J^+_\mu W^{+\mu} + J^-_\mu W^{-\mu}\right), \qquad J^-_\mu = \bar{e}_L\gamma_\mu\nu_{eL} + \bar{\mu}_L\gamma_\mu\nu_{\mu L} + \cdots$$

The interaction is purely left-handed — the weak force's defining 'V−A' chirality:

$$P_L = \frac{1-\gamma^5}{2}, \qquad \text{weak vertex} \;\propto\; \gamma^\mu(1-\gamma^5) \quad (V\!-\!A)$$

Written out from the doublet kinetic term, the charged current is forced once the doublets and the gauge coupling g are given:

$$\bar{L}_\ell\, i\hbar c\, \gamma^\mu D_\mu L_\ell \;\supset\; -\frac{g}{\sqrt{2}}\Big[\bar{\nu}_{\ell L}\gamma^\mu \ell_L\, W_\mu^+ + \bar{\ell}_L \gamma^\mu \nu_{\ell L}\, W_\mu^-\Big]$$

This is the key matching statement: if the reduction supplies the Standard Model SU(2)_L lepton doublets and gauge coupling, the W-boson charged current is not an additional postulate — it follows.

A.3 Muon decay and the low-energy Fermi limit

Muon decay proceeds through virtual W^- exchange: the muon emits a W^- and becomes ν_μ; the W^- becomes e^- and ν̄_e. The amplitude is

$$\mathcal{M} \sim \left(\frac{g}{\sqrt{2}}\right)^2 \frac{1}{q^2 - m_W^2}\left[\bar{u}_{\nu_\mu}\gamma^\mu P_L u_\mu\right]\left[\bar{u}_e \gamma_\mu P_L v_{\bar{\nu}_e}\right]$$

At the muon's energy the momentum transfer is far below the W mass, so the W propagator collapses to a contact term, defining the low-energy Fermi constant:

$$q^2 \ll m_W^2: \quad \frac{1}{q^2 - m_W^2} \simeq -\frac{1}{m_W^2} \quad \Longrightarrow \quad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$$

and the interaction becomes Fermi's local four-fermion theory:

$$\mathcal{L}_{\text{Fermi}} = -\frac{G_F}{\sqrt{2}}\left[\bar{e}\,\gamma^\mu(1-\gamma^5)\nu_e\right]\left[\bar{\nu}_\mu\gamma_\mu(1-\gamma^5)\mu\right] + \text{h.c.}$$

A.4 The W mass and the decay rate

The W is heavy because electroweak symmetry breaking gives it a mass — in the corpus, through a Wilson-line winding playing the Higgs role:

$$m_W = \tfrac{1}{2}\, g\, v, \qquad \mathcal{L} \supset m_W^2\, W_\mu^+ W^{-\mu}$$

With the effective interaction fixed, the three-body phase-space integral (neglecting the electron mass and radiative corrections) gives the muon decay rate:

$$\Gamma_\mu = \frac{G_F^2\, m_\mu^5}{192\pi^3}$$

— the inverse of the measured 2.2-microsecond lifetime. Route two has reproduced route one.

A.5 The general electroweak limit, and beta decay

Muon decay is the simplest case of a general statement. If the thirteen-dimensional geometry reduces to the four-dimensional electroweak action with SU(2)_L × U(1)_Y gauge fields, chiral doublets, electroweak breaking, and minimal coupling, then its low-energy charged-current sector is the ordinary weak interaction:

$$S_4^{\mathrm{EW}} = \int d^4x \left[-\tfrac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \tfrac{1}{4} B_{\mu\nu} B^{\mu\nu} + \sum_f \bar{\psi}_f \, i\hbar c \, \gamma^\mu D_\mu \psi_f + |D_\mu H|^2 - V(H) + \mathcal{L}_{\mathrm{Yuk}} \right]$$

$$SU(2)_L \times U(1)_Y \;\to\; U(1)_{\mathrm{em}}, \qquad m_W = \tfrac{1}{2} g v, \quad m_Z = \tfrac{1}{2}\sqrt{g^2 + g'^2}\, v$$

The same vertex, applied to quarks, drives nuclear beta decay; at low energy the down-to-up charged current becomes the beta-decay interaction, now dressed by hadronic form factors (g_V, g_A) because the nucleons are strong-force bound states:

$$\mathcal{L}_{\mathrm{CC}}^{\mathrm{quark}} = -\frac{g}{\sqrt{2}}\, \bar{u}_L \gamma^\mu V_{ud}\, d_L\, W^+_\mu + \mathrm{h.c.}$$

$$\xrightarrow{q^2 \ll m_W^2} \;\; \mathcal{L}_\beta \sim -\frac{G_F V_{ud}}{\sqrt{2}} \left[\bar{p}\, \gamma^\mu (g_V - g_A \gamma^5) n\right] \left[\bar{e}\, \gamma_\mu (1 - \gamma^5) \nu_e\right]$$

A.6 What rests where, and what remains conditional

It is worth separating clearly what is established in the source, what is completed here, and what is still owed.

— **What the corpus establishes directly.** The reduction of the thirteen-dimensional geometry to the electroweak sector — SU(2)_L from the internal geometry's spin-cover, U(1)_Y hypercharge, chiral left-handed doublets with no mirror partners (the anomaly witness 3·(1/6) − 1/2 = 0), and electroweak breaking giving massive W and Z bosons — together with the low-energy Fermi reduction and the relation $G\_F/\sqrt{2} = g^2/8m\_W^2$, carried as a derived identity that breaks against measurement if the geometry is wrong. All at 'interface' confidence: a stated, conditional reduction that recovers the textbook weak limit and does not replace the full Standard Model.

— **What is completed in this note.** The standard final step once the Fermi interaction is in hand: assembling the muon-decay amplitude and integrating the three-body phase space to $\Gamma\_\mu = G\_F^2\ m\_\mu^5/192\pi^3$. This is a textbook consequence, not separately written out in the corpus.

— **What remains conditional.** The full source-level obligation is to make the 13D → 4D electroweak reduction explicit and complete: (1) SU(2)_L and (2) U(1)_Y gauge fields; (3) chiral left-handed doublets and (4) right-handed singlets, with (5) no mirror weak sector; (6) the charge operator $Q = T_3 + Y$; (7) electroweak breaking with (8) massive W and Z; (9) canonical gauge-field normalization; (10) minimal coupling; (11) the relation $G\_F/\sqrt{2} = g^2/8m\_W^2$; and (12) no extra light charged weak boson. If all twelve hold, the chain — electroweak action → W charged current → integrate out W → Fermi theory → muon (and beta) decay — follows.

Closed result. Given the corpus's reduction of the 13-dimensional geometry to the 4-dimensional electroweak sector — $SU(2)_L \times U(1)_Y$ gauge fields, chiral lepton doublets, electroweak breaking with a massive W, and the derived Fermi relation $G_F/\sqrt{2} = g^2/8m_W^2$ — integrating out the W gives the same Fermi interaction and the same muon decay law, $\Gamma_\mu = G_F^2 m_\mu^5/192\pi^3$, with neutron beta decay the same charged current applied to quarks. This is a necessary low-energy consistency check, conditional on the twelve reduction conditions above — not a proof of the full construction.

Notes and sources

[1] The electroweak reduction — $SU(2)_L$ from the internal spin-cover, $U(1)_Y$ hypercharge, chiral doublets, electroweak breaking, and the Fermi/G_F relation — is in the source corpus, Paper II ('Forces'), §8 (the weak module, with §8.6.11.7 the Fermi limit and §8.6.15 the G_F identity): physics.magflowmeters.com/articles/Forces.html

[2] The electroweak scale v = 246.02 GeV, which sets the W mass, is a Paper I output (the Wilson-line determinant, Gate 8), imported rather than re-derived in the weak module — which is why this note checks the structure of the decay, not a from-nothing numerical lifetime.

[3] The by-hand electroweak anomaly witness $3\cdot(1/6) - 1/2 = 0$ — the fastest hand-check that the chiral matter content balances — is in Paper II, §8.6.4.

[4] The muon decay rate $\Gamma_\mu = G_F^2 m_\mu^5/192\pi^3$, the V−A structure, and integrating out the W are standard; see any quantum-field-theory or particle-physics text.

[5] Companion notes: *The Sun as a Lens* (gravity) and *Hydrogen as a Quantum Electrostatic Check* (electromagnetism), same series.