Hydrogen as a Quantum Electrostatic Check

How a shape built to explain particles also binds the electron in hydrogen
by exactly the textbook Coulomb rule — and how you can check it

A mostly non-technical note for anyone who loves quantum physics

— written to be checked by specialists, and read, equations and all, by everyone else —

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Before we start

If you can teach Coulomb's law on Monday and solve the hydrogen atom on Tuesday, this note is for you. The claim is deliberately small: if the thirteen-dimensional shape built to explain nature's particles really contains the Standard Model's forces, then its low-energy electromagnetic corner must reduce to ordinary quantum electrostatics — the same Coulomb potential, the same Schrödinger problem, the same hydrogen you already know. The cleanest test is hydrogen itself. Below we compute hydrogen the standard way, then show the same Coulomb potential and the same energy levels falling out of the thirteen-dimensional reduction, and place the two answers side by side.

Two honest words first. This is a **consistency check**, not a proof of the whole construction — passing it is a hurdle every candidate theory must clear, not evidence that the larger idea is right. And the shape itself is **not derived here**; we test one corner of it. You can read every word skipping every equation; the math is set off for anyone who wants to check it.

1. Where this check fits

This note is the electrostatic twin of a companion note, *The Sun as a Lens*, which checked the same geometry's gravity. The two checks run in parallel:

gravity check: 13D geometry → 4D Einstein gravity → the bending of starlight

electrostatic check: 13D geometry → 4D electromagnetism → the Coulomb force that binds hydrogen

What is being checked is narrow and concrete: the leading, low-energy quantum-electrostatic limit — a charged particle, the Coulomb potential, and the hydrogen energy levels. What is *not* being checked is everything finer: the tiny QED loop corrections (the Lamb shift), the hyperfine splittings, the full sweep of atomic spectroscopy, or the renormalization machinery of the complete Standard Model. Those are real and important, and none of them is claimed here. The single question is whether the simplest electrostatic quantum system comes out right.

2. Route one: hydrogen the standard way

Hydrogen is the right test because it is the simplest quantum system governed by electrostatics: one proton, one electron, one Coulomb attraction between them. It is familiar enough to draw on a classroom board and exact enough to pin down in a research lab.

The standard route is the one in every textbook. Gauss's law gives the electric field of the proton, and hence its potential; an electron of charge −e sitting in that potential has potential energy

$$V(r) = -\, e^2 / (4\pi\varepsilon_0\, r).$$

Drop that potential into the Schrödinger equation for the electron — using the electron–proton reduced mass μ — and the bound-state problem becomes

$$H = -(\hbar^2/2\mu)\nabla^2 - e^2/(4\pi\varepsilon_0\, r),$$

whose solution is the famous ladder of hydrogen energy levels,

$$E\_n = -\, 13.6 \text{ eV} / n^2, \quad \text{with ground-state size } a_0 \approx 0.529 \text{ Å (the Bohr radius).}$$

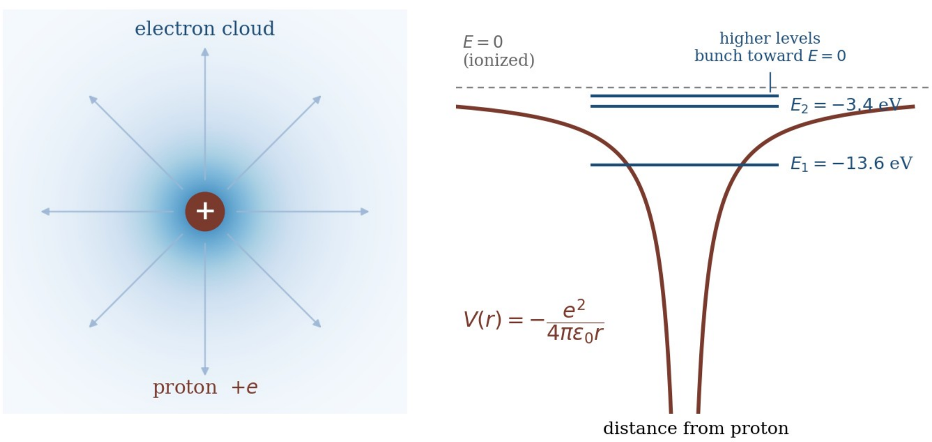

Figure 1. *The standard route. Gauss's law gives the proton's Coulomb potential (right); the electron occupies quantized levels in that well, E_n = −13.6 eV/n², with the ground state spread over the Bohr radius (left).*

This is the answer measured in every spectroscopy lab. Call it route one.

3. Route two: the same hydrogen from the geometry

Now the second route — the same Coulomb potential and the same energy levels, this time emerging from the thirteen-dimensional shape rather than assumed.

The shape's tiny internal space carries the Standard Model's forces as its low-energy vibrations. After electroweak symmetry breaking — the same process that gives the W and Z particles their mass in ordinary physics — one combination of those vibrations stays massless: the photon, the carrier of electromagnetism. The shape also fixes how strongly each particle couples to that photon, which is just its electric charge, given by the charge operator $Q = T_3 + Y$. That rule returns −1 for the electron and +1 for the proton — exactly the observed values.

Once you have a photon and the rule for charge, the rest is standard physics, carried out here rather than assumed. Charged matter couples to the photon in the one way consistency allows (‘minimal coupling’), which is the same coupling that defines ordinary quantum electrodynamics. Take the static limit and the photon’s equation becomes Gauss’s law again, giving the very same Coulomb potential $\varphi(r) = e/(4\pi\varepsilon_0 r)$. Take the slow-particle limit of the relativistic electron equation and it becomes the same Schrödinger problem. So the geometry hands you the identical Hamiltonian

$$H = -(\hbar^2/2\mu)\nabla^2 - e^2/(4\pi\varepsilon_0\, r),$$

and therefore the identical hydrogen spectrum. The two routes meet.

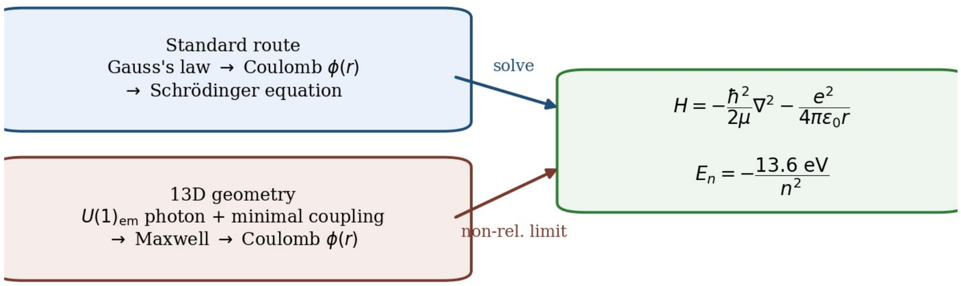

Figure 2. *Two routes, one Hamiltonian. The standard route assumes the photon and the Coulomb law; the 13-dimensional reduction supplies them — a massless U(1) photon and minimal coupling — and the same low-energy electromagnetic action gives the same electrostatic problem, hence the same spectrum.*

As with the gravity check, this isn’t circular. The shape was selected by requirements about particles and forces that never mentioned hydrogen or the Coulomb rule. That the same shape, reduced, reproduces the exact electrostatic attraction binding the electron is a construction built for entirely different reasons landing, unprompted, on the right answer.

The full reduction — the gauge action, the minimal coupling, the static limit, and the slow-particle reduction of the relativistic electron equation — is written out in Appendix A, along with exactly what the source corpus establishes directly and what is completed in this note.

4. The two routes meet — and not just hydrogen

Step by step, the two routes line up on every ingredient of the problem:

Quantity	Standard quantum electrostatics	13D reduction route
Photon field	assumed as the EM gauge field	unbroken $U(1)_{em}$ zero mode after electroweak breaking
Charge	empirical ±e	charge operator $Q = T_3 + Y$
Field equation	Gauss's law	Maxwell equation from the reduced action
Potential	$\varphi = e/(4\pi\varepsilon_0 r)$	*same*
Electron potential energy	$V = -e^2/(4\pi\varepsilon_0 r)$	*same*
Quantum equation	Schrödinger equation	nonrelativistic limit of minimally coupled Dirac/QED
Hydrogen spectrum	$E_n = -13.6\ \mathrm{eV}/n^2$	*same*

Once the Hamiltonian is the same, the spectrum is the same — there is nothing left to compute. And hydrogen is only the simplest case. The same reduction supplies the photon and the charge rule for all charged matter at once, so the whole low-energy electrostatic world comes along: every Coulomb attraction and repulsion between charged particles, and the many-body electrostatic Hamiltonian that underlies the electrostatics of atoms and molecules. Hydrogen is the front door; behind it is ordinary quantum electrostatics in general.

5. What this does, and does not, show

To keep the claim the right size: reproducing the Coulomb potential and the hydrogen spectrum is something *any* candidate theory has to do. Passing means the construction cleared a hurdle it genuinely could have failed — the photon might have come out massive, the charges wrong, or the potential modified at short range — but clearing it does not prove the larger construction. It earns a closer look.

What it does not show, said plainly: it does not prove the full thirteen-dimensional construction; it does not derive the Lamb shift or the hyperfine structure; it does not compute the spectra of the whole periodic table; it does not derive the measured strength of electromagnetism (the fine-structure constant α) from nothing, beyond whatever the companion corpus separately claims; and it does not replace the loop calculations of full QED. It is a necessary low-energy consistency check, and only that.

The real evidence for the construction lives where it matches numbers it was never built to match — the structure of the forces, the three families of matter, the particle properties that follow from a handful of measured inputs. This check is one more door that opens rather than slamming shut.

6. The seam, now closed

As with the gravity note, the step from ‘the geometry contains electromagnetism’ to ‘therefore hydrogen comes out right’ is written out in full in Appendix A. In one breath: the shape’s electroweak sector leaves a massless photon and the charge rule $Q = T_3 + Y$; minimal coupling plus the static limit reproduce Gauss’s law and the Coulomb potential; and the slow-particle limit of the relativistic electron equation reproduces the Schrödinger Hamiltonian — hence $E_n = -13.6\ \mathrm{eV}/n^2$. You don’t need the appendix to follow the story; it’s there so anyone — including an AI you ask to check it — can verify each step.

What the corpus establishes directly is the electromagnetic reduction itself, carried at ‘interface’ confidence: the unbroken U(1) photon after electroweak breaking, the charge operator $Q = T_3 + Y$ with its $\mathbb{Z}_6$ charge quantization, and the static Coulomb/Gauss limit. What this note completes are the standard quantum-mechanical steps that follow once that gauge sector is in hand — the minimal coupling, the Dirac-to-Schrödinger reduction, and the hydrogen solution. And what remains genuinely conditional is named, not hidden: the reduction must yield exactly one massless photon (no extra light photon-like force), the canonically normalized Maxwell action, the observed charges, and no extra correction large enough to bend Coulomb’s law at distances already measured. If all of those hold, the hydrogen result follows.

Appendix A — The full chain, from the 13-dimensional gauge sector to the hydrogen spectrum

This appendix writes out, step by step, the derivation summarized in Section 6. It is not required reading for the story; it is here so the claim can be checked. The goal is narrow: show that the thirteen-dimensional geometry's electromagnetic subsector reduces to ordinary quantum electrostatics — the Coulomb potential and the hydrogen Schrödinger problem. Conventions follow the source corpus: E = −∇φ with ∇·E = +ρ/ε_0, so the minus sign sits on φ, not on Gauss's law.

A.1 The electromagnetic sector from the 13-dimensional geometry

The thirteen-dimensional construction carries the Standard Model gauge fields as the low-energy zero modes of its internal geometry. For this check, isolate the electromagnetic subsector after electroweak symmetry breaking, SU(2)_L × U(1)_Y → U(1)_em. One combination of gauge fields stays massless — the photon — and the electric charge of each field is set by the charge operator Q = T_3 + Y:

$$A_\mu^{\mathrm{em}} = \sin\theta_W\, W_\mu^3 + \cos\theta_W\, B_\mu, \qquad Q = T_3 + Y$$
$$Q_e = -1, \qquad Q_p = +1$$

The relevant low-energy degrees of freedom are then the photon A_μ, the electron field ψ_e, and the proton (effective) field ψ_p, governed by the reduced four-dimensional electromagnetic action:

$$S_{\mathrm{EM}} = \int d^4x \left[-\frac{1}{4\mu_0} F_{\mu\nu}F^{\mu\nu} + \bar{\psi}_e\big(i\hbar c\,\gamma^\mu D_\mu - m_e c^2\big)\psi_e + \bar{\psi}_p\big(i\hbar c\,\gamma^\mu D_\mu - m_p c^2\big)\psi_p \right]$$

with the covariant derivative carrying the charge of each field through minimal coupling:

$$D_\mu = \partial_\mu + \frac{iq}{\hbar}A_\mu, \qquad q_e = -e, \qquad q_p = +e$$

The geometry's role is precisely this: it supplies the photon field and the charge operator. The four-dimensional action then supplies standard minimal coupling — the same coupling that defines QED — so charged matter interacts through the same electromagnetic potential A_μ as in ordinary quantum electrodynamics.

A.2 The electrostatic limit: Gauss's law and the Coulomb potential

Take the electrostatic limit: fields time-independent, magnetic and radiative effects neglected, so A_i ≈ 0 and A_0 = φ. The photon's equation of motion for A_0 reduces to Gauss's law:

$$\nabla^2\phi = -\frac{\rho}{\varepsilon_0} \qquad (\text{static fields},\ A_i \approx 0,\ A_0 = \phi)$$

For a proton modeled as a point charge at the origin, this integrates to the Coulomb potential:

$$\rho(\mathbf{x}) = e\,\delta^3(\mathbf{x}) \quad \Longrightarrow \quad \phi(r) = \frac{e}{4\pi\varepsilon_0 r}$$

and an electron of charge q_e = −e coupled minimally to it has potential energy:

$$V(r) = q_e\,\phi(r) = -\frac{e^2}{4\pi\varepsilon_0 r}$$

— identical to the standard route’s Coulomb potential.

A.3 The nonrelativistic limit: from Dirac to Schrödinger

Now take the nonrelativistic limit of the relativistic (Dirac) equation for the electron with minimal coupling. In the low-velocity, positive-energy limit it reduces to the Pauli–Schrödinger Hamiltonian:

$$\big(i\hbar\gamma^\mu D_\mu - mc\big)\psi = 0$$
$$\xrightarrow{\text{non-relativistic}} \quad H = \frac{1}{2m}\big(-i\hbar\nabla - q\mathbf{A}\big)^2 + q\phi - \frac{q\hbar}{2m}\,\boldsymbol{\sigma}\cdot\mathbf{B} + \cdots$$

In the electrostatic hydrogen limit (A = 0, B = 0, q = −e, with φ the Coulomb potential), and using the electron–proton reduced mass μ for the two-body problem, this becomes:

$$(\mathbf{A} = 0,\ \mathbf{B} = 0,\ q = -e) \quad \Longrightarrow \quad H = -\frac{\hbar^2}{2m_e}\nabla^2 - \frac{e^2}{4\pi\varepsilon_0 r}$$
$$\xrightarrow{\text{two-body},\ \mu} \quad H = -\frac{\hbar^2}{2\mu}\nabla^2 - \frac{e^2}{4\pi\varepsilon_0 r}$$

which is exactly the Hamiltonian of the standard route.

A.4 The hydrogen spectrum

The bound-state problem is then the textbook one,

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 - \frac{e^2}{4\pi\varepsilon_0 r}\right]\psi = E\psi, \qquad \mu = \frac{m_e m_p}{m_e + m_p}$$

whose solution is the hydrogen spectrum and Bohr radius:

$$E_n = -\frac{\mu e^4}{2(4\pi\varepsilon_0)^2\hbar^2}\,\frac{1}{n^2} = -\frac{13.6\ \mathrm{eV}}{n^2}, \qquad a_0 = \frac{4\pi\varepsilon_0\hbar^2}{\mu e^2} \approx 5.29\times 10^{-11}\ \mathrm{m}$$

— the same numbers measured in every spectroscopy lab. Route two has reproduced route one.

A.5 Not just hydrogen: the general electrostatic limit

Hydrogen is only the simplest case. The same reduction supplies the photon and the charge rule for every charged field at once, so the general statement follows: if the thirteen-dimensional geometry reduces to the four-dimensional Standard Model gauge action with the correct unbroken U(1)_em, charge operator, and minimally coupled matter, then its low-energy electrostatic quantum limit is ordinary quantum electrostatics —

$$S_{13} \;\longrightarrow\; S_4 \;\supset\; \int d^4x \left[-\frac{1}{4\mu_0}F_{\mu\nu}F^{\mu\nu} + \sum_f \bar{\psi}_f\big(i\hbar c\,\gamma^\mu D_\mu - m_f c^2\big)\psi_f\right]$$

$$H_{\mathrm{es}} = \sum_i \left(-\frac{\hbar^2}{2m_i}\nabla_i^2\right) + \sum_{i<j} \frac{q_i q_j}{4\pi\varepsilon_0\,|\mathbf{x}_i - \mathbf{x}_j|}$$

— the many-body electrostatic Hamiltonian underlying the electrostatics of atoms and molecules.

A.6 What rests where, and what remains conditional

It is worth separating clearly what is established in the source, what is completed here, and what is still owed.

— **What the corpus establishes directly.** The reduction of the thirteen-dimensional gauge/matter geometry to the four-dimensional electromagnetic sector — the unbroken U(1)_em photon after electroweak breaking, the charge operator $Q = T_3 + Y$ with its $\mathbb{Z}_6$ charge quantization, and the static Coulomb/Gauss limit ($\nabla^2\varphi = -\rho/\varepsilon_0$, $E = Q/4\pi\varepsilon_0 r^2$). This is carried at ‘interface’ confidence: a stated, conditional reduction that recovers the static Coulomb/Gauss limit and explicitly does not replace QED or Maxwell.

— **What is completed in this note.** The standard quantum-mechanical steps that follow once that gauge sector is in hand: minimal coupling, the nonrelativistic Dirac-to-Schrödinger reduction, and the hydrogen bound-state solution. These are textbook consequences, not separately derived in the corpus.

— **What remains conditional.** The full source-level obligation is to make the 13D → 4D QED reduction explicit and complete: (1) exactly one unbroken U(1)_em photon; (2) the charge operator $Q = T_3 + Y$; (3) a canonically normalized Maxwell kinetic term; (4) minimal coupling to charged matter; (5) the observed electron and proton charges; (6) no extra light photon-like gauge boson; and (7) no scalar or vector correction large enough to modify Coulomb's law at tested distances. If all seven hold, the chain — 4D QED action → Gauss's law → Coulomb potential → nonrelativistic Schrödinger limit → quantum electrostatics — follows.

> **Closed result.** Given the corpus's reduction of the 13-dimensional geometry to the 4-dimensional electromagnetic gauge sector — one massless $U(1)_{em}$ photon, charge operator $Q = T_3 + Y$, and the static Coulomb/Gauss limit — minimal coupling and the nonrelativistic limit reproduce the Coulomb potential $V(r) = -e^2/4\pi\varepsilon_0 r$ and the hydrogen Hamiltonian $H = -(\hbar^2/2\mu)\nabla^2 - e^2/4\pi\varepsilon_0 r$, hence $E_n = -13.6$ eV/n^2 and $a_0 \approx 0.529$ Å. This is a necessary low-energy consistency check, conditional on the seven reduction conditions above — not a proof of the full construction.

Notes and sources

[1] The electromagnetic reduction — unbroken $U(1)_{em}$, $Q = T_3 + Y$, the $\mathbb{Z}_6$ charge rule, and the static Coulomb/Gauss limit — is in the source corpus, Paper II ('Forces'), §8.6 (the charge convention's home) and §9.6 (the QED/electrostatics ladder): physics.magflowmeters.com/articles/Forces.html

[2] A fully hand-checkable electrostatic consistency calculation — the Coulomb field energy of a charged shell, worked two ways to six significant figures — is in Paper I ('GUT'), Appendix O.

[3] The hydrogen solution (Coulomb potential, Schrödinger equation, $E_n = -13.6$ eV/n^2, Bohr radius) is standard and appears in every introductory quantum-mechanics text.

[4] The nonrelativistic reduction of the Dirac equation to the Pauli–Schrödinger Hamiltonian is standard; see any graduate quantum-mechanics or quantum-field-theory text.

[5] Companion note: *The Sun as a Lens* — the gravitational consistency check in the same series.